A Study on Multivariate Quality-Control Charts

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CERTIFICATE

This is to certify that the present work contained in the thesis entitled "**A Study on Multivariate Quality-Control Charts**" has been carried out by **Ms. Sheela Kumari (Roll No. 9811424)** under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

Many problems in industrial quality control involve a vector of measurements of several characteristics simultaneously rather than a single characteristics For example, we may want to simultaneously control both the length and the diameter of the pipe In other words, both the length and diameter must be acceptable for the pipe to be usable Although one could monitor the process by separate control chart for each quality characteristics, but results of monitoring depend on the correlation between characteristics. If the correlation is high, the result may not be acceptable. So, controlling each characteristic separately may not yield a product in which all variables should be acceptable In many industries, separate control chart for each variable are used due to computational intricacies of multivariate control charts But, now with modern computers, one can easily use several types of multivariate charts. In this thesis, the performance of different charts (some of which are multivariate charts and some are univariate charts) are computationally evaluated at different correlation coefficient, sample-size and process mean or process variance deviations levels Comparison is also done at different levels of three factors For different charts, different factors and their interaction are significant

Performance of charts which are basically for detecting process mean deviation or process variance deviation are also examined when process variance deviation or process mean deviation is introduced The present study thus provided some preliminary findings, which indicate that correlation coefficient, sample size and process mean or process variance deviation have effects on the performance of multivariate quality-control charts.

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SHEELA KUMARI

Date:

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INTRODUCTION

There are many situations in which the simultaneous monitoring or control of two or more related quality characteristics is necessary For example, suppose that a bearing has both an inner diameter (x_1) and an outer diameter (x_2) that together determine the usefulness of the part Suppose that x_1 and x_2 have independent normal distributions As both quality characteristics are measurements, they could be monitored by applying the usual $\bar{x}$ chart to each characteristic The process is considered to be in control only if the sample means $\bar{x}_1$ and $\bar{x}_2$ fall within their respective control limits

Monitoring these two quality characteristics independently can be very misleading The probability that either $\bar{x}_1$ or $\bar{x}_2$ exceeds 3-sigma control limits is 0 0027 However, the joint probability that both variables exceed their control limits simultaneously when they are both in control is (0.0027)(0 0027) = 0 00000729, which is considerably smaller than 0.0027 Furthermore, the probability that both $\bar{x}_1$ and $\bar{x}_2$ will simultaneously plot inside the control limits when process is really in control is (0 9973)(0.9973) = 0 99460729 Therefore, the use of two independent $\bar{x}$ charts has distorted the simultaneous monitoring of $\bar{x}_1$ and $\bar{x}_2$, in that the type I error and the probability of a point correctly plotting in control are not equal to their advertised levels for the individual control charts.

This distortion in the process monitoring procedure increases as the number of quality characteristic increases. In general, if there are p-statistically independent quality characteristic for a particular product and if X_Bar chart with P (type 1 error) = α is maintained on each, then the true probability of type 1 error for the joint control procedure is

$$\alpha' = 1 - (1 - \alpha)^p \tag{1.1}$$

and the probability that all p means will simultaneously plot inside their control limits when the process is in control is

$$P\{\text{all } p \text{ means plot in control}\} = (1-\alpha)^p \qquad 1.2$$

Clearly, the distortion in the joint control procedure can be severe, even for moderate values of p Furthermore, if the p quality characteristics are not independent, which usually would be the case if they relate to the same product, then equation-(1) and (1 2) do not hold, and we have no easy way even to measure the distortion in the joint control procedure

A Study on Multivariate Quality-Control Charts is done in the present work In the next section problem definition, literature-review and the scope of present work are discussed

1.1 Problem Definition

Process monitoring problems in which several related variables are of interest are called **multivariate quality control problems.** These processes can not be measured by using univariate control charts due to above-mentioned problem. Usually in the industries the multivariate processes are handled by using univariate control charts due to computational and mathematical intricacies. But now with the help of modern computers, multivariate control charts can be used with ease

In this thesis attempt is made to study different types of multivariate quality-control charts with their performance evaluation also.

1.2 Literature Review

The original work in multivariate quality was done by **Hotelling (1947),** who applied his procedure to bombsight data during World War II. He suggested the use of the statistics

$$T^2 = n\left(\bar{x} - \bar{\bar{x}}\right)' S^{-1} \left(\bar{x} - \bar{\bar{x}}\right)$$

A major disadvantage of Hotelling T2 statistics is that it does not indicates which variable or set of variables is out-of-control, when there is signal

Mason, Tracy, Young (1995) had provided the approach for identifying the source of signals given by a T2 statistics in a multivariate control chart They decompose the T2 statistics into independent parts, each of which is similar to an individual T2 variate, each of which reflects the contribution of an individual variable Each T2 is compared with some F distribution value and based on this, decisions are taken

Jackson (1980, 1991) discusses decomposing the T2 statistics into a sum of p principal components and using these components for identifying the out_of_control variable

He decomposed Σ_0 as $\Sigma_0 = C\Lambda C'$ where C is the matrix of eigenvectors and Λ the diagonal matrix of eigenvalues, the vector of scaled principal components corresponding to case X is given by $U = \Lambda^{-1/2}C'(X - \mu_0)$ Since $T^2 = U'U$, so U is a decomposition of T2 into independent N (0,1) components

When the principal components represent meaningful groupings of variables, the interpretation of out_of_control signals is readily apparent However, in many cases it is difficult or even impossible, to attach meaning to the principal components, and the characteristics associated with the out_of_control signals can not be determined

Jackson and Mudholkar (1979) also recommended a procedure in which the leading principal components of each case are computed and to predict the case

Nedumaran and Pignatiello Jr. (1996) have also proposed approach for diagnosing signals from T^2 and χ^2 control charts They proposed that once a signal is given on a T^2 or a χ^2 chart, univariate diagnostic charts are constructed for the *p variables*, any meaningful linear combination of the *p* variables and their *p* principal components in such a way that the overall probability of false alarm is controlled at the specified value α' for all the univariate control charts. When a signal is issued by a T^2 or a χ^2 chart, statistics from the signaling subgroup are plotted on the 2*p-univariate* diagnostic charts to

identify the variables and/or the principal components that caused the signal Thus the T^2 or a χ^2 chart is used to monitor the multivariate process, whereas the univariate diagnostic charts are used only for diagnosing signals issued by the T^2 or a χ^2 chart

Doganaksoy, Tucker and Faltin (1991) suggested ranking the p variables based on their univariate t-statistics and find the P-value for each of the p variables when the null hypothesis of no change in their respective means is tested If the P-value of any variable is smaller than the corresponding Bonferroni error rate α' / p, one is to conclude that the mean of that variable has changed This process is similar to using p univariate control charts with limits $\overline{\overline{x}}_k \pm t_{df,\alpha/2p'} \sqrt{s_{kk} / n}$ where $\overline{\overline{x}}_k$ and s_{kk} are the standard mean and deviation of kth variable $t_{df,\alpha'/2p}$ is the $(1-\alpha'/2p)$ th percentile point of a t-distribution with df degrees of freedom, and n is the subgroup size

According to Hayter and Tsui **(1994),** a "good" solution to the multivariate control problem is that the procedure should satisfy the following three conditions

- Control the overall familywise error rate at the nominal level α
- Provide a simple mechanism for detecting which variables contributed to the out_of_control condition.
- Quantify the magnitude of the mean change for the out_of_control variables

They suggested the procedure besed on the construction of exact simultaneous confidence intervals for each of the variables means that meet each of these three goals. Both parametric and nonparametric procedures are considered.

In their approach, a signal is issued for the multivariate process when an *M statistic* exceeds the critical values $C_{R,\alpha}$ The M statistics is defined by

$$M = \max_{1 \le i \le p} \frac{|X_i - \mu_i|}{\sigma_i}$$

where X_i is the current observation of the ith variable, and $\mu_i and \sigma_i$ are the mean and the standard deviation of the ith variable, respectively When a signal is given, one has to examine each of the p univariate control charts with control limits set at $\mu_i \pm C_{R,\alpha}\sigma_i$ for diagnosing the signal

Nedumaran and Pignatiello Jr. (1996)' approach for diagnosing signal given by the T^2 or a χ^2 chart identified the variables or the principal components responsible for the signals even in the cases where the **Hayter and Tsui (1994),** method failed to signal

Holmes and Mergen (1993) have also proposed technique for out-of-control signal on the T^2 or χ^2 chart

They proposed to construct standardized value for each of the p variables when there is signal The standardized values are then compared with control limits of $0 \pm z_{\alpha/2}$

The limits are calculated using the Bonferroni approach which says that the effective value of α (i e , Type I error) risk (eff α) is approximately

eff $\alpha = p\alpha$,

Where α is the risk associated with each variable

This method is different from the other methods in that capability standard deviation is estimated using the mean square successive difference (MSSD) Also standardized value are plotted simultaneously on the same display

Timm, N.H. (1996) suggested Finite Intersection Test (FIT), in which they use single step and stepdown FIT to evaluate whether a multivariate process is in control or out_of_control The FIT computer program is also developed by Cox, Frag, Boudreau, and Timm (1994) .

The stepdown FIT procedure provides an alternative to the procedure proposed by the **Hayter and Tsui (1994),** the sequential test procedure by the **Jackson and Bradley (1996),** and regression adjustment procedure of **Hawkins (1991, 1993).** The procedure is

more powerful than **Roy's** stepdown recommended by **Wierda (1994b)** and the procedure of **Mason, Tracy, and Young (1995)**

When multivariate normality does not hold, the stepwise resampling method of multiple hypothesis listing developed by **Troendle (1995)** may be used. **Troendle's** procedure incorporates the structure of the covariance matrix into the problem without making distribution assumptions However, it does not provide simultaneous confidence interval for population parameters

In multiple processes, a Shewhart chart is often used at each individual step If the steps of processes are independent then this is a meaningful procedure However, when steps are correlated, it is difficult to interpret **Hotelling** T^2 can be used here, but process quality characteristics should be normal random variables Also, when the process is out-of-control, it is difficult to say, which variable is responsible

Zhang (1980, 1982, 1984, 1985a, 1985b, 1989a, 1989b, 1992) had proposed a different approach for this **Zhang (1984)** call his chart "cause-selecting chart", which is similar to in concept to the regression control chart of **Mandal (1969)** in that a control chart is constructed for a variable only after the observation have been adjusted for the effects of some other random variable Cause-selecting charts use incoming quality measurements ant out-going quality measurements in an attempt to distinguish between in-coming quality problems and problems in the current operation It has advantage over the multivariate T^2 chart in that it provides diagnostic information regarding which subprocess is out of control Also it handles complicated relationship between the variables.

Wade and Woodall (1993) propose using prediction limits with cause-selecting chart to improve their statistical performance

When false-alarm rates of the cause-selecting chart using prediction limits and **Zhang**'s limits were examined, it was found that the false-alarm rate of **Zhang**'s limits depends on

the sample size foe estimating the linear relationship Thus the advantage of using prediction limits is that the false-alarm rate is not affected by the sample-size

Another chart using T^2 is that of **Alwan (1986),** who proposed a cumulative sum (CUSUM) of the T^2

Crosier (1988) suggested a CUSUM of T (COT) But CUSUN of T2 is preferred over CUSUM of T due to computational simplicity

Crosier also proposed another CUSUM scheme, refereed to as multivariate CUSUM, forms a CUSUM vector directly from the observations CUSUM vector is not based on the T^2 of the individual observation, but its signals are based on a Hotelling T^2 applied to the CUSUM vectors

Both procedures allow use of recent enhancements for the CUSUM schemes, such as FIR feature Both of this CUSUM procedure reduces to multivariate Shewhart charts when the CUSUM scheme parameter h is 0

Another approach to multivariate CUSUM control is the proposal of **Woodall and Ncube (1985).** This consists of a set of individual CUSUM's of the variables of X, and a signal is given if any of the p CUSUM's exceeds its control limit This approach has the better interpretability, since signal obtained corresponds to an immediately identified variable But, when variables are highly correlated, then this gives poor result

In the case of strongly correlated variables, they suggested the use of principal components for improvement of the performance Though signal is given for a component rather than a particular variable, there may be some loss of interpretability in term of the original variables.

Pignatiello and Runger (1990) also proposed two multivariate CUSUM charts

First multivariate CUSUM scheme is based on the following vectors of cumulative sums

$$D_i = \sum_{j=i-l_i+1}^{i} X_j$$

and

$$MC_t = \max\left\{0, \left(D_t'\Sigma^{-1}D_t\right)^{1/2} - k_2 l_t\right.$$

where $k_2 > 0$ and

$$l_t = l_{t-1} + 1 \qquad \text{if } MC_{t-1} > 0$$
$$= 1, \qquad \text{otherwise,}$$

I = 1,2 ,3, an out-of-control signal is given as sonn as $MC_t > h_3$ where h_3>0

Second CUSUM chart is the CUSUM of Hotelling T^2

First multivariate CUSUM scheme of **Crosier** is similar to this,but yet distinctively different from this CUSUM vector scheme

Crosier "contracts" or "shrinks" each of his vectors of cumulative sums toward the zero vector by multiplying the cumulative sum by a scalar The norm of the contracted cumulative sum is then compared with an upper control limit The ARL performance of **Crosier'** multivaiiate cumulative sum chart is similar to that of **Pignatiello and Runger's.**

Woodall and Ncube' multiple univariate CUSUM scheme and **Pignatiello and Runger's** CUSUM vector scheme give roughly the same performance when the process experience a mean shift in just one variable or characteristic But when means of several variables shifts simultaneously to off-target values, univariate CUSUM chart gives poorer result then CUSUM vector scheme

Lowry, Woodall, Rigdon (1992) proposed multivariate extension of the univariate exponentially weighted moving average (EWMA) chart The ARL performance shows that MEWMA procedure can perform better than the multivariate CUSUM procedure of **Crosier (1988)** and **Pignatiello and Runger (1990)** when the process is initially out-of-control and it performs roughly the same if the shift in the mean vector is delayed.

Healy (1987) provided a general multivariate framework to produce more powerful diagnostics than those based on the T^2 statistics

Suppose it is anticipated that when the process mean shifts from μ to μ_A Then the optimal test statistic for a shift in the ith measured vector is the scalar quantity

$$(\mu_A - \mu)' \Sigma^{-1} (X_i - \mu) = (\mu_A - \mu)' Z_i$$

where the p x 1 vector Z_i is defined by

$$Z_i = \Sigma^{-1} (X_i - \mu)$$

where vector Z_i is actually a vector of regression-adjusted variables

The ith component of the Z_i is the standardized residual resulting when X_i is regressed on all other variables in X

Hawkins (1991) put **Healy's** result and **Mandal (1969)** regression adjustment together and proposed a different approach

1.3 Scope of Present Work

The present work focuses on the following issues

- To study the effect of correlation coefficient between different variables, sample size and deviation in the process mean or process variance on the different charts
- To study the effect of mean deviation on the charts meant for measuring process variance deviation and the effect of variance deviation on the charts meant for measuring process mean deviation

1.4 Organization of The Thesis

Chapter 2 entitled **'Description of Multivariate Quality-Control Charts'** discusses different control charts for measuring process mean and variance deviation It also discusses the merits and demerits of these charts

Chapter 3 entitled **'Performance Evaluation'** covers two issues namely performance of different charts with respect to correlation coefficient between different variables, sample size and deviation in the process mean or process variance on the different charts and the comparison of different charts

Chapter 4 entitled **'Results and Conclusions'** summarizes all the findings that have been made through out the work and the conclusion that follow from these findings

DESCRIPTION OF MULTIVARIATE QUALITY-CONTROL CHARTS

This chapter discusses different types of quality control charts Some charts are only for monitoring process mean changes and some are only for monitoring variance changes Both types of charts are discussed Their merits and demerits are also focused

Charts for Monitoring Mean Changes in The Multivariate Processes:

- CHI-SQUARE (χ^2) CONTROL CHART
- T_SQUARE (T^2) CONTROL CHART
- MULTIVARIATE CUSUM CHARTS :
  (1) CUMULATIVE SUM OF T
  (2) CUSUM VECTOR SCHEME
- MULTIVARIATE EXPONENTIALLY MOVING-AVERAGE CHART
- REGRESSION-ADJUSTMENT PROCEDURE
  (1) Z-METHOD FOR MONITORING PROCESS MEAN
  (2) Y-METHOD
- UNIVARIATE SHEWHART X-BAR
- UNIVARIATE X-CUSUM CHART FOR MONITORING PROCESS MEAN

Charts for Monitoring Variance Changes in the Multivariate Processes:

- REGRESSION-ADJUSTMENT PROCEDURE
  1. Z-METHOD FOR MONITORING PROCESS VARIANCE
- UNIVARIATE X_RANGE CHART
- MULTIVARIATE PROCESS VARIANCE CHART
- UNIVARIATE S^2 CONTROL CHART

- UNIVARIATE X-CUSUM CHART FOR MONITORING PROCESS VARIANCE

2.0 MONITORING MEAN CHANGES IN MULTIVARIATE PROCESSES :

2.1 CHI-SQUARE (χ^2) CONTROL CHART

Assumption:

The joint probability distribution of the p quality characterisrtic is the p variate normal distribution

This procedure requires computing the samples mean for each of the p quality characteristics from a sample of size n This set of quality characteristic mean is represented by px1 vector

$$\overline{x} = \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \\ \\ \overline{x}_p \end{bmatrix}$$

Where α is the confidence level

The test statistics plotted on the Chi-Square Chart for each sample is

$$\chi^2 = n(\overline{x} - \mu)' \Sigma^{-1} (\overline{x} - \mu) \qquad 2\,1\,1$$

Where $\mu' = [\mu_1, \mu_2, \; ..., \mu_p]$ is the vector of in-control means for each quality characteristics and Σ is the co-variance matrix The upper limit on the control chart is

$$UCL = \chi^2_{\alpha,p} \qquad 2\,1\,2$$

Estimating μ and Σ

In practice, it is usually necessary to estimate μ and Σ from the analysis of preliminary sample of size n, taken when the process is assumed to be in control If m such samples are available the sample means and variances are calculated from each sample as follows

$$\overline{x_{jk}} = \frac{1}{n}\sum_{i=1}^{n} x_{ijk} \qquad \begin{cases} j = 1, 2, \quad , p \\ k = 1,2, \quad , p \end{cases} \qquad 2\ 1\ 3$$

$$S_{jk}^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_{ijk} - \overline{x_{jk}})^2 \qquad \begin{cases} j = 1,2, \quad , p \\ k = 1,2, \quad , p \end{cases} \qquad 2\ 1\ 4$$

Where x_{ijk} is the ith observation on the jth quality characteristics in the kth sample. The covariance between quality characteristic j and quality characteristics h in the kth sample is

$$S_{jhk} = \frac{1}{n-1}\sum_{i=1}^{n} (x_{ijk} - \overline{x_{jk}})(x_{ihk} - \overline{x_{hk}}) \qquad \begin{cases} k = 1,2, \quad , m \\ j \neq h \end{cases} \qquad 2.1.5$$

The statistics $\overline{x_{jk}}, S_{jk}^2$ and S_{jhk} are then averaged all over m samples to obtain

$$\overline{\overline{x}}_j = \frac{1}{m}\sum_{k=1}^{m} \overline{x_{jk}} \qquad j = 1,2, \ldots, p \qquad 2.1\ 6$$

$$\overline{S_j^2} = \frac{1}{m}\sum_{k=1}^{m} S_{jk}^2 \qquad j = 1,2, \quad , p \qquad 2.1\ 7$$

and

$$\overline{S_{jh}} = \frac{1}{m}\sum_{k=1}^{m} S_{jhk} \qquad j \neq h \qquad 2\ 1\ 8$$

The $\overline{\overline{x}}_j$ are the elements of the vector $\overline{\overline{\mathbf{x}}}$, and the p x p averages of sample covariance matrices **S** is formed as

$$S = \begin{bmatrix} \overline{S_1^2} & \overline{S_{12}} & \overline{S_{13}} & & \overline{S_{1p}} \\ & \overline{S_2^2} & \overline{S_{23}} & & \overline{S_{2p}} \\ & & \overline{S_3^2} & & \\ & & & & \\ & & & & \overline{S_p^2} \end{bmatrix} \qquad 2.1.9$$

The averages of the sample covariance matrices **S** is an unbiased estimate of Σ when the process is in control

2.2 T-SQUARE (T^2) CONTROL CHART :

Assumption:

When the process is in control, the X_i are independent and follow a multivariate normal distribution with mean vector μ and covariance matrix Σ

If we replace μ with $\overline{\overline{x}}$ and Σ with S in Eq-2.1.1, test statistics now becomes

$$T^2 = n\left(\overline{x} - \overline{\overline{x}}\right)' S^{-1} \left(\overline{x} - \overline{\overline{x}}\right) \qquad 2.2.1$$

In this form , the procedure is called the **Hotelling** T^2 **Control Chart**

The control limit for T^2 control chart are given by

$$UCL = \frac{p(m-1)(n-1)}{mn-n-p+1} F_{\alpha,\, p,\, mn-m-p+1} \qquad 2.2.2$$

$$LCL = 0$$

2.2.1 The Case Where n = 1

In some settings the subgroup size is generally n = 1

Example : chemical industries

Suppose that m samples, each of size n=1 are available, and that p is the number of quality characteristics observed in each sample Let x and S be the sample mean vector and covariance matrix, respectively, of these observations The Hotelling T^2 statistics in Eq-2 2 1 becomes

$$T^2 = (x - \bar{x})' S^{-1} (x - \bar{x}) \qquad 2\,2.3$$

The control limits are given by

$$UCL = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha,\,p,\,m-p} \qquad 2\,2\,4$$

$$LCL = 0$$

when the number of sample m is large , say m > 100 , upper control limits is given by

$$UCL = \frac{p(m-1)}{m \;\; - p} F_{\alpha,p,m-p} \qquad 2\,2\,5$$

or

$$UCL = \chi^2_{\alpha,\,p} \qquad 2\,2\,6$$

2.2.2 Merits and Demerits T^2 or χ^2 Control charts

Hotelling T^2 and χ^2 control charts are two of the most popular multivariate control chart, because of its similarity to a univariate Shewhart control chart It is often called

multivariate Shewhart- type control chart **These charts are widely used when the observations are independent and identically distributed p-variate normal random variables** The probability of incorrectly declaring the process to out of control can be maintained at any desired level in these charts

These are the optimal test statistics for detecting a general shift in the process mean vector for an individual multivariate observation, but it is not optimal for more structured mean shifts --- for example when shifts in only some of the variables

The major drawback of this chart is that they are unable to predict which variable of set of variables is out-of-control Also mean shifts are always confounded with variance shifts, which require quite extensive analysis following a signal to determine the nature of the shifts These procedures are based on only the most recent observation, it is insensitive to small and moderate shifts in the mean vector

2.2.3 Identifying The Sources For Out-of-Control Signals When The T^2 or χ^2 Control Chart is Used:

Difficulties encountered with any of the multivariate control chart are practical interpretation of an out-of-control signal Which of the variable of (set of variables) is responsible for signal is difficult to say The standard practice is to plot **univariate** $\bar{x}$ charts on the individual variables $x_1, x_2, \ldots, x_p$ But this approach may not be successful due to correlation of variables

T^2 or χ^2 value which is out-of-control indicates that one or more of the mean has changed or that the correlation between two or more of the variables has changed, but its difficult to determine which variable is out-of-control

Following method is one of the recommended methods for identifying the out-of-control variable

Donald S. Holem and A. Erhan Mergen (1993) had used Bonferroni-type control limits The method is done in two steps as follows

In first step, we calculate the standardized value for each of the p variables. The standardization is done like this

$$z_{ij} = \frac{x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}}{s_j} \qquad 2.2.3.1$$

Where m is the number of samples. s_j is the mean square successive difference estimator for the capability standard deviation. It is defined as

$$s_j = \left(\frac{\sum_{i=1}^{n-1} (x_{i+1} - x_i)^2}{2(n-1)} \right)^{\frac{1}{2}} \left(1 - \frac{3}{8n}\right)^{-1} \qquad 2.2.3.2$$

In second step, we make a control chart type of display of the standardized values. The limits on the display are calculated using the Bonferroni approach which says that the effective value of the α (i.e., type I error) risk (effective α) is approximately

$$\text{eff } \alpha = p\alpha \qquad 2.2.3.3$$

where α is the risk associated with each variable.
The approximate z for the control limits is established using α = eff α/p. Then the limits are set at $0 \pm z_{\alpha/2}$

Example:
When p = 2 and eff α = 0.003, the corresponding α and z values 0.0015 and the $z_{0.0015/2} = 3.19$ respectively. The variable plotting outside this control limit is considered responsible for signal. If all standardized values fall within control limits, the correlation

of variables may changed This method is different from the other methods in following aspects The first difference is that we estimate the capability standard deviation using the mean square successive difference MSSD approach The second difference is the simultaneous plotting of the standardized values on the same display

2.3 MULTIVARIATE CUSUM CHARTS :

The most frequent application of the CUSUM schemes is the detection of change in the mean of a normally distributed variable

Ronald B. Crosier (1988) had given two-multivariate cumulative sum (CUSUM) quality procedure The first CUSUM procedure reduces each multivariate observation to a scalar and then forms a CUSUM of scalars The second CUSUM procedure forms a CUSUM vector directly from the observations

2.3.1 CUSUM # 1 :

The most direct method is to form a CUSUM of scales T_n (n = 1,2,3,) This method is called COT scheme

The CUSUM is given by

$$S_n = \max(0, S_{(n-1)} + T_n - k) \qquad 2\,3\,1\,1$$

where $S_0 \geq 0$ and k>0

T_n (Square root of Hotelling T^2) is given by

$$T_n = +\left[(X_n - \mu)' \Sigma^{-1} (X_n - \mu)\right]^{1/2} \qquad 2\,3.1\,2$$

The COT scheme signals when $S_n > h$ (Upper Control Limit)

The COT scheme offers several advantages over multivariate Shewhart charts First COT scheme may be designed to give any false alarm rates by using desired type I error Second, recent enhancements for CUSUM scheme may be applied to COT schemes A robustness procedure, such as the two-in-a-row rule **(Lucas and Crosier (1982))** may be applied to the sequence of T statistics The two-in-a-row rule states that outliers should be ignored (i e not used in the CUSUM calculations) but the occurrence of two outliers in succession is taken as an off-aim signal The fast initial response (FIR) feature for CUSUM schemes **(Lucas and Crosier (1982))** is easily extended to COT schemes Quick detection of an initial off-aim condition is obtained by starting the scheme with S_0 equal to h/2 rather than 0

2.3.2 CUSUM #2 :

A vector valued CUSUM scheme is derived by replacing the scalar quantities of a univariate CUSUM scheme by vectors

The univariate CUSUM scheme is given by

$$S_n = \max(0, S_{(n-1)} + (X_n - \mu) - k\sigma) \qquad 2\,3\,2\,1$$

where μ is the target value for the mean, σ is the standard deviation of the X's, k>0 and $S_0 = 0$

Replacing the scalars by vectors gives $s_n = \max(0, s_{n-1} + (x - \mu) - k)$.

But the problem is then, how to find k and how to interpret taking the maximum of vector and the null vector In the univariate case, the quantity $S_{(n-1)} + (X_n - \mu)$ is shrunk toward 0 by k standard deviation But for multivariate case k must satisfy $k'\Sigma^{-1}k = k^2$, that is k must be of length k, where length is defined using the covariance matrix Σ If the subtraction of k is to shrink to $s_{n-1} + x_n - \mu$ toward 0, then k must be in the same

direction as $s_{n-1}+x_n-\mu$ Hence $k=(k/C_n)(s_{n-1}+x_n-\mu)$, where C_n is the length $s_{n-1}+x_n-\mu$ The maximum of null vector can be taken as $s_n=0$, rather than subtracting k, whenever the length of k is greater than the length of $s_{n-1}+x_n-\mu$, that is whenever k>Cn Rather than calculating the vector k and subtract it from $s_{n-1}+x_n-\mu$ It is simpler to just contract the vector $s_{n-1}+x_n-\mu$ by (1-k/Cn), provided that k<Cn Hence multivariate vector CUSUM method is expressed as

$$C_n=\left[(s_{n-1}+x_n-\mu)'\Sigma^{-1}(s_{n-1}+x_n-\mu)\right]^{1/2} \qquad 2\ 3\ 2\ 2$$

then

$$s_n=0 \qquad \text{if } C_n\le k \qquad 2\ 3\ 2\ 3$$

$$s_n=(s_{n-1}+x_n-\mu)(1-k/C_n) \qquad \text{If } C_n>k \qquad 2\ 3\ 2\ 4$$

where $s_0=0$ and k>0

$$Y_n=\left[s_n'\Sigma^{-1}s_n\right]^{1/2} \qquad 2\ 3\ 2\ 5$$

The multivariate CUSUM vector scheme signals when Y_n> h

The multivariate CUSUM vector scheme give quicker detection of small shifts than both multivariate Shewhart charts and COT schemes when n = 1. But, for n>1, COT method gives better result than Cusum vector method. The multivariate CUSUM chart also don't give the indication that which variable is out-of-control when there is signal

2.4 MULTIVARIATE EXPONENTIALLY WEIGTHED MOVING-AVERAGE CONTROL CHART :

The exponentially weighted moving-average (or EWMA) control chart is also a good alternative to the Shewhart chart when we are interested in detecting small shifts The performance of the EWMA control chart is approximately equivalent to that of CUSUM chart, and in some ways easy to set up and operate It is also commonly used with sample size n=1

The univariate EWMA chart is based on the values

$$Z_i = rX_i + (1-r)Z_{i-1} \qquad 2\,4\,1$$

i = 1,2, where $Z_0 = \mu_0 = 0$ and $0 < r \le 1$

If Xi are independent $N(0,\sigma^2)$ random variables, then the mean of Z_i is 0 and the variance is

$$\sigma^2_{Z_i} = \{r[1-(1-r)^{2i}]/(2-r)\}\sigma^2,\ \mathrm{I} = 1,2,$$

Thus when the in control value of the mean is 0, the control limits of the EWMA chart are often set at $\pm L\sigma_{Z_i}$, where L and r are the parameters of the chart

In the multivariate case, a natural extension is to define vectors of EWMA's

$$Z_i = RX_i + (I-R)Z_{i-1} \qquad 2\,4\,2$$

I = 1,2,3, ,where $Z_0 = 0$ and $R = diag(r_1, r_2, \ldots r_p), 0 < r \le 1$, j=1,2,3, ,p

The MEWMA chart gives out-of-control signal when

$$T_i^2 = Z_i' \Sigma_{Z_i}^{-1} Z_i > h \qquad 2\,4\,3$$

where h (>0) is chosen to achieve a specified incontrol ARL and Σ_{Z_i} is the covariance matrix of Z_i

Σ_{Z_i} is given by

$$\Sigma_{Z_i} = \sum_{j=1}^{i}\left[R(I-R)^{i-j}\Sigma(I-R)^{i-j}R\right] \qquad 2.4.4$$

if $r_1 = r_2 = \quad = r_p \; = r$, then

Σ_{Z_i} is given by

$$\Sigma_{Z_i} = \left\{r\left[1-(1-r)^{2i}\right]/(2-r)\right\}\Sigma \qquad 2.4.5$$

if there is no a priori reason to weight past observations differently for the p quality characteristics being monitored,then $r_1 = r_2 = \quad = r_p \; = r$

if $r_1 = r_2 = \quad = r_p \; = r$, then the MEWMA vectors can be written as

$$Z_i = rX_i + (1-r)Z_{i-1} \qquad 2.4.6$$

Smaller values of r are more effective in detecting small shifts in the mean vector

The ARL performance of MEWMA chart is similar to_that_of multivariate CUSUM chart. But when n > 1, we find earlier signal in multivariate CUSUM chart than MEWMA. This chart also doesn't have provision for identification of out-of-control variables

2.5 REGRESSION ADJUSTMENT FOR VARIABLES IN MULTIVARAIATE QUALITY CONTROL :

Problems in multivariate quality problems occur when one observes, not a single characteristics, but a vector of characteristics It is of course possible to monitor the individual characteristics by charting them separately, but to extent that they are correlated. It can be more effective to monitor a vector of characteristics in some

multivariate way The main point of regression adjustment is that the nature of this multivariate focus should, where possible, take into account the characteristics mechanisms that lead to the association between the process measures Two important possible mechanisms are the following

- A mechanism that leads to some or all of the measurement being related, but where possible for any of them to undergo parameter shifts without affecting the others

  Example:

  measurements of different dimensions of switch drum it is possible here that one dimension can shift ,without affecting others But response to problem can be speeded up by appropriate use of the measures on the other dimensions

- A mechanisms that leads to measures having a natural ordering in which if any measures undergoes a parameter shift it may affect some or all of the measures following it, but none of the measures preceding it in this ordering This sort of ordering is called a cascade And the resulting vector of measurements described as having the cascade property

  Cascade property is typified by serial value-added manufacturing processes

Example:

A metallurgical flotation plant where the material passes through successive enrichment steps The quality of product at the end of any steps depends not only on that step is operating correctly, but also on how well the preceding steps performed If a particular step goes out of control, it can affect product measures, not just at that step, but also some or all of the following steps

Only the first mechanism is discussed in this thesis.

For the first mechanism, **Hawkins (1991)** defines a set of regression – adjusted variables using the vector

$$Z = \left[diag\left(\sum_{0}^{-1} \right) \right]^{-\frac{1}{2}} Y = A(X - \mu_0) \qquad 2\ 5\ 1$$

where the transformation matrix A is given by

$$A = \left[diag\left(\sum_{0}^{-1} \right) \right]^{-\frac{1}{2}} \sum_{0}^{-1} \qquad 2\ 5\ 2$$

Z_i is the standardized residual when the ith variable is regressed on the remaining p-1 variables in X

In the second type of mechanism, **Hawkins** decomposes the T^2 statistic using the standardized residuals from the regression of the ith variable on the first i-1 variables These are defined by

$$Y = C(X - \mu) \qquad 2\ 5\ 3$$

where C is the Cholesky lower triangular root of Σ^{-1}

2.5.1 Improvement in Performance Due to Regression Adjustment

The diagnostics Y and Z have two motivations One is that they provide controls on the scale of the individual measures and the other is that they are more sensitive to out-of-control situations because the regression adjustment reduces the background variability, letting the shifts stand out more clearly The relative performance of regression-adjusted and unadjusted diagnostics depends on the degree of correlation on which the regression is based Suppose that on the original scale of the mean of X_j shifts by δ standard deviations and that an adjustment is made by regressing X_j on the variable that affect it Let this regression have a multiple correlation coefficient of R_j The regression-adjusted

variable will also have its mean shifted by δ standard deviation of X_j, but since the regression residuals have a standard deviation smaller by a factor of

$$\sqrt{1-R_j^2}$$

This shift will be

$$\frac{\delta}{\sqrt{1-R_j^2}}$$

of the regression-adjusted variable's standard deviations At one extreme,if Rj is zero,then there is no improvement in performance over working on the x scale At the other extreme ,if Rj is close to 1,the margin of improvement of the regression-adjusted variable over the original variable can be arbitrarily large

This is also important in regression adjustment is that, the variable for which regression-adjustment is made, they did not undergo any shift in mean in mean If they did, then the calculated improvement in performance would not occur This has implications for the type of regression adjustments made Suppose that measurements have cascade property Then it is correct to adjust each X_j for $X_1, \quad , X_{j-1}$ but not for any of $X_{j+1}, \quad , X_p$ (i e one should use the diagnostics Y_j) Suppose Z_j are erroneously used instead, so that X_j is regression-adjusted not only for $X_1, \quad , X_{j-1}$, but also for $X_{j+1}, \quad , X_p$, which are they affected by the shift in X_j Then the regression-adjustment would incorrectly attenuate the mean shifts so that performance of Z_j would not be as good as that using Y_j and perhaps not even as good as that using X_j itself

2.5.2 REGRESSION ADJUSTMENT METHOD FOR TYPE 1 MECHANISM :

Assumptions:

- When the process is in control, the $X \sim N(\mu,\Sigma)$ are independent and follow a multivariate normal distribution with mean vector μ and covariance matrix Σ

While the process is in control, $\mu = \mu_0$ and $\Sigma = \Sigma_0$

- All the components of X are rescaled to unit standard deviation—that is, that the diagonal elements of Σ_0 are equal to 1

Suppose σ_{ij} and σ^{ij} and are the i, jth elements of Σ_0 and Σ_0^{-1} respectively

The multiple regression of X_i, the ith component of X, on all other components of X is written as

$$X_i - \mu = \Sigma_{j \neq i} \beta_{ij} (X_j - \mu_j) + \varepsilon_i \qquad 2\ 5\ 2\ 1$$

τ_{ii} = residual variance i e the variance of the ε_i

In terms of variance and covariance

$$\tau_{ii} = \sigma_{ii} - \Sigma_{j \neq i} \beta_{ij} \sigma_{ij}$$

$$2\ 5\ 2.2$$

$$\sigma^{ii} = -\beta_{ij} / \tau_{ii} (i \neq j)$$

Instead of completely unspecified shift in μ, assume that the out-of-control affects the mean of a single variable i, shifting the process distribution to $N(\mu_0 + \delta e_i, \Sigma_0)$, where e_i is a column vector whose value is 1 in row i and 0 elsewhere and the scalar δ is the magnitude of the shift then the liklihood-ratio statistic for the hypothesis $\delta = 0$ is given by

$$Z_i = \left[(X_i - \mu) - \sum_{j \neq i} \beta_{ij} (X_i - \mu) \right] \Big/ \tau_{ii}^{1/2} \qquad 2\ 5\ 2\ 3$$

Whose null distribution is N(0,1) This (scalar) Z_i is the residual when X_i is regression all other components of X rescaled to unit variance

The vector of scaled residual is also expressed as

$$Y_i = \sum_0^{-1}(X - \mu_0) \qquad 2\ 5\ 2\ 4$$

From Equations-2 5 2 4, the ith component of Y is the regression residual when variable I is regressed on all other variables, scaled by a factor τ_{ii}^{-1}. Under control, $Y \sim$ N (0, Σ_0^{-1}), Z is just rescaling of Y

$$Z = \left[\ diag\left(\sum_0^{-1}\right)\right]^{-1/2} Y = A(X - \mu_0) \qquad 2\ 5\ 2.5$$

where the transformation matrix

$$A = \left[diag\left(\sum_0^{-1}\right)\right]^{-1/2} \sum_0^{-1} \qquad 2\ 5\ 2\ 6$$

The coefficients of A provide an instant visual diagnostics of the relative effectiveness of multivariate control using Z over univariate control using X If A is close to an identity matrix, then there will be little magnification of mean shifts in going from X to z, so the proposed methods will not be noticeably more effective even for single-variable shifts than univariate control methods Against this, numerically large elements in A (particularly its diagonals) imply large magnifications of mean shifts and large potential benefits from using this method The matrix A also shows the effects of changes in more than one elements of μ_0 A vector shifts of say Δμ in the mean would translate into a vector of shifts in Z of AΔμ, which is easily evaluated

It follows from definition that Z ~ N (0,B), where

$$B = \left[diag\left(\sum_{0}^{-1} \right) \right]^{-\frac{1}{2}} \sum_{0}^{-1} \left[diag\left(\sum_{0}^{-1} \right) \right]^{-\frac{1}{2}} \qquad 2\,5\,2\,7$$

The relationship between X, Z and T^2 is

$$T^2 = n\left(\bar{x} - \bar{\bar{x}}\right)' S^{-1}\left(\bar{x} - \bar{\bar{x}}\right)$$
$$= \sum_{0}^{-p} (X - \mu_0) Y_i \qquad 2\,5\,2\,8$$

or, rewriting in terms of Z,

$$T^2 = \sum_{i=1}^{p} W_i$$

Where $W_i = (X_i - \mu_i) Z_i \tau_{ii}^{-\frac{1}{2}}$ 2.5 2 9

The vector Z provides the possibility of separate control chart of the individual variables in X , thus gives trigger for the out-of-control variable also

The major advantage of Z_method is that we don't need separate diagnostic procedure to know about the responsible variables for out-of-control sample But when we run the process even after out-of-control signal, the signals get mixed with other variable also, and undeviated variable also starts to give out-of-control signal Since one is usually interested both in special causes affecting the particular case and possible persistent shifts in the process,it is generally desirable to use combined Shewhart/CUSUM control charts In this thesis Shewhart X-bar, X-Range chart, X-Cusum for mean and scale for individual X's and Z-Cusum chart for mean and scale for Z elements have been used

2.5.2.1 Cusum Control Chart for Z elements :

CUSUM control of location and scale is given by as follows:

$$
\begin{aligned}
L_{i0}^{+} &= L_{i0}^{-} = S_{i0}^{+} = S_{i0}^{-} = 0 \\
W_{ni} &= (|Z_{ni}|^{1/2} - 0\ 822)/0\ 349 \\
L_{in}^{+} &= \max(0,\ L_{i,n-1}^{+} + Z_{ni} - k) \\
L_{in}^{-} &= \min(0,\ L_{i,n-1}^{-} + Z_{ni} + k) \\
S_{in}^{+} &= \max(0,\ S_{i,n-1}^{+} + W_{ni} - k) \\
S_{in}^{-} &= \min(0,\ S_{i,n-1}^{-} + W_{ni} + k)
\end{aligned}
\qquad 2\ 5\ 2\ 1\ 1
$$

The decision –interval CUSUM's L^{+} and L^{-} test for shifts of location in the upward and downward direction, respectively The CUSUM's S^{+} and S^{-} test for shifts of variability in the upward and downward directions, respectively Their basis is the observation that if Z ~ N(0,1), then $|Z|^{1/2}$ closely approximates an N (0 822,0 349 x 0 349) distribution , and that changes in the scale of Z affect the location of $|Z|^{1/2}$ By plotting all four CUSUM's on a common chart, one obtains a powerful CUSUM control for location and scale An out-of-control is signaled if any of these four CUSUM quantities exceeds the decision interval h Along with this Cusum chart of mean and scale, Shewhart X_bar chart and X_Range chart are used for X's, which are described below

2.5.2.2 UNIVARIATE SHEWHART X-BAR AND X-RANGE CHARTS :

Assumption:

Quality characteristics are normally distributed with mean μ and standard deviation σ These charts are used in Z_method of regression adjustment procedure This chart is

developed by **Dr. Walter A. Shewhart (1920).** We usually do not know μ and σ, so we estimate them from past data, which are in control But, when we have standard value of μ and σ, we used these charts with standard value of μ and σ

Charts Based on Estimated Value of μ and σ:

X_bar chart:

If $x_1, x_2, \quad , x_n$ is a sample of size n, then the averages of this sample is

$$\bar{x} = \frac{x_1 + x_2 + \qquad + x_n}{n} \qquad 2\,5\,2\,2\,1$$

If $\overline{x_1}, \overline{x_2}, \overline{x_3}, \quad , \overline{x_m}$ are the averages of sample m, then best estimator of μ is given by

$$\bar{\bar{x}} = \frac{\overline{x_1} + \overline{x_2} + \qquad + \overline{x_m}}{m} \qquad 2.5\,2\,2.2$$

Control Limits for X_bar chart

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2 \bar{R} \\ \text{Center Line} &= \bar{\bar{x}} \\ \text{LCL} &= \bar{\bar{x}} - A_2 \bar{R} \end{aligned} \qquad 2\,5\,2.2\,3$$

Where A_2 is constant depends on sample size

Range Chart:

In this chart, we estimate standard deviation from ranges of the m samples The range of the sample is the difference between the largest and smallest observation in sample size n, that is

$$R = x_{\max} - x_{\min}$$

If $R_1, R_2, \quad . \quad , R_m$ be the ranges of the m samples. The average range is

$$\overline{R} = \frac{R_1 + R_2 + \qquad + R_m}{m} \qquad 2\,5\,2\,2\,4$$

Control Limits for Range chart:

$$\begin{aligned} \text{UCL} &= D_4 \overline{R} \\ \text{Center Line} &= \overline{R} \\ \text{LCL} &= D_3 \overline{R} \end{aligned} \qquad 2\,5\,2\,2\,5$$

The constants D_3 and D_4 depends on sample size n

Charts Based on Standard Values of μ And σ:

When we know the standard vales of μ and σ, we use these standard to establish the control charts for $\bar{x}$ and R without analysis of past data

Suppose that the standards for mean and standard deviation is μ and σ

Control Limits for X_bar chart:

$$\begin{aligned} \text{UCL} &= \mu + 3\frac{\sigma}{\sqrt{n}} \\ \text{Center line} &= \mu \\ \text{LCL} &= \mu - 3\frac{\sigma}{\sqrt{n}} \end{aligned} \qquad 2\,5\,2\,2\,6$$

Control Limits for Range chart:

$$\begin{aligned} \text{UCL} &= D_2 \sigma \\ \text{Center Line} &= d_2 \sigma \\ \text{LCL} &= D_1 \sigma \end{aligned} \qquad 2\,5\,2.2\,7$$

Where constants D_2, d_2, D_1 depend on sample size n

If the sample size is relatively small, the range method gives almost as good an estimator of the variance σ^2 as the sample variance S^2 For moderate values of n, say $n \geq 10$, the range method loses efficiency rapidly, as it ignores all the information in the sample between x_{max} and x_{min} So we use S control chart for sample size > 1

Control Charts for X_Bar and S:

$\bar{x}$ and S are used for all values of n as it gives good result then range method Earlier range method was used due to simplicity of calculations In these charts we calculate sample average $\bar{x}$ and the sample standard deviation S for each sample If σ^2 is the unknown variance of a probability distribution, then an unbiased estimator of σ^2 is the sample variance

$$S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

But the sample standard deviation S is not an unbiased estimator of σ Actually, S estimates $c_4\sigma$, where constant c_4 depends on sample size n

Charts Based on Standard values of μ and σ:

When we have standard value of μ and σ, we don't estimate it from past data

Control Limits for the $\bar{x}$ chart:

The parameters of $\bar{x}$ chart is given by Eq-2 5 2 2 6

Control Limits for the S Chart:

$$\begin{aligned} \text{UCL} &= B_6\sigma \\ \text{Center line} &= c_4\sigma \\ \text{LCL} &= B_5\sigma \end{aligned} \qquad 2.5\,2\,2\,8$$

Where constants B_6, c_4, B_5 depend on sample size n.

Charts Based on Estimated values of μ and σ:

If no standard is given for $\bar{x}$ and σ, we estimate it from past data Let S_i is the standard deviation of the ith sample The average of the m preliminary samples is

$$\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_i$$

Standard deviation σ is estimated as $\dfrac{\bar{S}}{c_4}$

Control Limits for the $\bar{x}$ chart:

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + \frac{3\bar{S}}{c_4\sqrt{n}} \\ \text{Center line} &= \bar{\bar{x}} \\ \text{LCL} &= \bar{\bar{x}} - \frac{3\bar{S}}{c_4\sqrt{n}} \end{aligned} \qquad 2\,5\,2\,2\,9$$

Control Limits for the S Chart :

$$\begin{aligned} \text{UCL} &= B_4\bar{S} \\ \text{Center line} &= \bar{S} \\ \text{LCL} &= B_3\bar{S} \end{aligned} \qquad 2\,5\,2\,2\,10$$

Where constants B_4 and B_3 depend on sample size n.

2.5.2.2.1 Control Charts for Individual Measurements :

When the sample size n =1, we do not use above mentioned $\bar{x}$ and S charts or $\bar{x}$ and Range charts Here, we use moving range concepts

The moving range MR_i is given as $|x_i - x_{i-1}|$, where x_i is the observation of ith sample

We establish control charts for monitoring $\bar{x}$ and σ based on moving range

Control Limits for the $\bar{x}$ Chart:

$$\text{UCL} = \bar{x} + 3\frac{\overline{MR}}{d_2}$$

$$\text{Center line} = \bar{x} \qquad 2\,5\,2\,2\,1\,1$$

$$\text{LCL} = \bar{x} - 3\frac{\overline{MR}}{d_2}$$

Constants d_2 is taken as value corresponding to sample size n = 2, since we substract consecutive observation

Here, we use moving range chart for monitoring variance deviation

Control Limits for Moving Range Charts:

$$\text{UCL} = D_4\overline{MR}$$

$$\text{Center line} = \overline{MR} \qquad 2\,5\,2\,2\,1\,2$$

$$\text{LCL} = D_3\overline{MR}$$

Here, also we take value of D_4 and D_3 corresponding to n = 2

2.5.2.3 UNIVARIATE X-CUSUM CHART :

This chart is also used in Z_method of regression adjustment procedure The major disadvantage of any Shewhart control chart is that it only uses the information about the

process contend in the last plotted point and it ignores any information given by the entire sequence of points Due to this reason, the Shewhart control chart is sensitive to small shifts in the process For this reason we use cumulative sum control chart of small sets Because it considers all the information in the sequence of sample values by plotting the cumulative sums of the deviation of the sample values from a target value

Cumulative-sums control charts were first proposed by **Page (1954).**

Assumption:

The quality characteristics are normally distributed with mean μ and standard deviation σ

If $x_1, x_2, \ldots, x_m$ are the individual observation of m samples, the standardized two-sided Cusum is given by

$$C_i^+ = \max\left[0, y_i - k + C_{i-1}^+\right]$$
$$C_i^- = \max\left[0, -k - y_i + C_{i-1}^-\right] \qquad 2.5\,2\,3\,1$$

where, $$y_i = \frac{x_i - \mu_0}{\sigma}$$

If sample size n >1, $$y_i = \frac{\overline{x_i} - \mu_0}{\sigma / \sqrt{n}}$$

and $C_0^+ = C_0^- = 0$

Where, $\overline{x_1}, \overline{x_2}, \ldots, \overline{x_m}$ are the averages of m samples of size n and and $C_0^+ = C_0^- = 0$

k = K/σ

K is called the reference value and is given by,

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2}$$

where, $\mu_1 = \mu_0 + \delta\sigma$ = deviated mean from target value

If either C_i^+ or C_i^- exceed the decision interval h, the process is called out of control

Due to standardization many Cusum charts can use the same value of k and h and these parameters do not depend on σ.

2 5.2.3.1 Cusum for Monıtoring Process Variability:

Cusum chart can also be used for monitorıng process varıabılıty

Hawkıns (1981)(1993a) has suggested a new standardızed quantıty

$$v_i = \frac{\sqrt{|y_i|} - 0\ 822}{0\ 349} \qquad 2.5\ 2\ 3\ 1\ 1$$

He suggests that the v_i are sensıtıve to varıous changes rather than mean changes But ın fact, they are sensıtıve to both mean and varıance changes. The ın-control dıstrıbutıon of v_i ıs N (0,1)

The two standardızed Cusum scale chart ıs defined as

$$S_i^+ = \max\left[0, v_i - k + S_{i-1}^+\right] \qquad 2\ 5\ 2\ 3.1\ 2$$

$$S_i^- = \max\left[0, -k - v_i + S_{i-1}^-\right] \qquad 2\ 5\ 2\ 3\ 1\ 3$$

where $S_0^+ = S_0^- = 0$

2.6 MONITORING PROCESS VARIABILITY IN MULTIVARIATE PROCESSES :

Two methods are also analyzed for monıtorıng process varıabılıty

- **Method Based on Sample Generalized Variance**
- **Univariate S^2 Control Chart**

2.6 1 Sample Generalized Variance Method:

This method is used for monitoring multivariate process variability In multivariate process, process variability is summarized by the $p \times p$ covariance matrix Σ, which is estimated by sample covariance matrix S, which is given by

$$S = \begin{bmatrix} \overline{S_1^2} & \overline{S_{12}} & \overline{S_{13}} & \overline{S_{1p}} \\ & \overline{S_2^2} & \overline{S_{23}} & \overline{S_{2p}} \\ & & \overline{S_3^2} & \\ & & & \\ & & & \overline{S_p^2} \end{bmatrix} \qquad 2\,6\,1\,1$$

Sample covariance matrix S, is calculated by analysis of in-control preliminary data This method is based on the sample covariance matrix $|S|$

Control Limits for Chart:

$$\begin{aligned} \text{UCL} &= |\Sigma|(b_1 + 3b_2^{1/2}) \\ \text{Center line} &= b_1|\Sigma| \qquad 2\,6\,1\,2 \\ \text{LCL} &= |\Sigma|(b_1 - 3b_2^{1/2}) \end{aligned}$$

If we estimate Σ from sample covariance matrix S, then we take $|S|/b_1$ as estimator of Σ. b_1 and b_2 are given by

$$b_1 = \frac{1}{(n-1)^p}\prod_{i=1}^{p}(n-i) \qquad 2\,6.1\,3$$

$$b_2 = \frac{1}{(n-1)^{2p}} \prod_{i=1}^{p} (n-i) \left[\prod_{j=1}^{p} (n-j+2) - \prod_{j=1}^{p} (n-j) \right] \qquad 2.6\ 1\ 4$$

We calculate the sample covariance matrix for each sample similar to Eq-2.6 1 1 and values of $|S_i|$ is plotted for each sample on the control chart with above mentioned control limits

This method shows drawbacks, when determinant of matrices are equal, even though elements of matrices are different i e they have different process variability and correlation between the variables Hence, it is wise to use univariate S^2 control charts in conjunction with this chart

PERFORMANCE EVALUATION

This chapter is organized into two sections In the first section, computational performance of all charts are discussed which is based on the output obtained from the respective charts code. In the second section design of experiments are discussed for all chart Since the performance of charts seems to depend on the correlation coefficient, sample size and deviation in the process mean or process deviation, factorial design of experiment are performed for all charts and their performance at different levels of these three factors are analyzed

3.1 Computational Performance

Many problems in industrial quality control involve a vector of measurements of several characteristics simultaneously rather than a single characteristics For example, we may want to simultaneously control both the length and the diameter of the pipe In other words, both the length and diameter must be acceptable for the pipe to be usable Although one could monitor the process by separate control chart for each quality characteristics, but results of monitoring depend on the correlation between characteristics If the correlation is high, the result may not be acceptable. So, controlling each characteristic separately may not yield a product in which all variables should be acceptable In many industries, they use separate control chart for each variable due to computational intricacies of multivariate control charts But, now with modern computers, one can easily use several types of multivariate charts The performance of different charts (some of which are multivariate charts and some are univariate charts) are computationally evaluated at different correlation coefficient, sample-size and deviations levels Univariate charts are assumed not to give good results, when there is high correlation between variables So, they are analyzed at different correlation coefficient to observe the relationship. The performances of different multivariate charts are also evaluated to see the effect of three parameter (Correlation coefficient, sample size and deviations)

Following charts for monitoring mean changes in the multivariate processes are analyzed:

- CHI-SQUARE (χ^2) CONTROL CHART
- T_SQUARE (T^2) CONTROL CHART
- MULTIVARIATE CUSUM CHARTS :
  - (1) CUMULATIVE SUM OF T
  - (2) CUSUM VECTOR SCHEME
- MULTIVARIATE EXPONENTIALLY MOVING-AVERAGE CHART
- REGRESSION-ADJUSTMENT PROCEDURE
  - (1) Z-METHOD FOR MONITORING PROCESS MEAN
  - (2) Y-METHOD
- UNIVARIATE SHEWHART X-BAR
- UNIVARIATE X-CUSUM CHART FOR MONITORING PROCESS MEAN

Following charts for monitoring process variance changes in the multivariate processes are analyzed:

- REGRESSION-ADJUSTMENT PROCEDURE
  1. Z-METHOD FOR MONITORING PROCESS VARIANCE
- UNIVARIATE X_RANGE CHART
- MULTIVARIATE PROCESS VARIANCE CHART
- UNIVARIATE S^2 CONTROL CHART
- UNIVARIATE X-CUSUM CHART FOR MONITORING PROCESS VARIANCE

Codes are developed for all charts. Since original data (measurements of quality characteristics of multivariate process) are not available, so test data are generated from multivariate normal random generator The subroutine for multivariate normal random generator is available in the **NAG** library (written in **FORTRAN** language) Two sets of problems are designed, one for two variables and another

for five variables Performance of these charts is evaluated for different values of correlation coefficient, sample size and deviation in process mean or variance

Values of Factors :

Following values for different factors are selected

Values of Correlation Coefficient Two Variables case:

- **Low value**

1.000000	0.100000
0.100000	1.000000

- **Medium value**

1.000000	0.723009
0.723009	1.000000

- **High value**

1.000000	0.900000
0.900000	1.000000

Values of Correlation Coefficient for Five Variables case :

- **Low value**

1.000000	0.100000	0.100000	0.100000	0.100000
0.100000	1.000000	0.100000	0.100000	0.100000
0.100000	0.100000	1.000000	0.100000	0.100000
0.100000	0.100000	0.100000	1.000000	0.100000
0.100000	0.100000	0.100000	0.100000	1.000000

- **Mixed value**

1.000000	0.138800	0.349600	0.082900	0.265200
0.138800	1.000000	0.732400	0.913000	0.693200
0.349600	0.732400	1.000000	0.682400	0.821400
0.082900	0.913000	0.682400	1.000000	0.764000
0.265200	0.693200	0.821400	0.764000	1.000000

- **High value**

1.000000	0.900000	0.900000	0.900000	0.900000
0.900000	1.000000	0.900000	0.900000	0.900000
0.900000	0.900000	1.000000	0.900000	0.900000
0.900000	0.900000	0.900000	1.000000	0.900000
0.900000	0.900000	0.900000	0.900000	1.000000

Sample Sizes :

Low (1), medium (6) and High (10)

Deviations :

Deviation for process means and variance are measured as multiple of process variance. Combinations of 0.25, 0.5, 1.0, 1.5, and 2.0 of process variance (σ)) both positive and negative were used to generate deviation from the mean and variance.

Initially, using NAG library subroutine sixty base data are generated, which are in-control for all charts Then, specified deviation is given after 50^{th} sample and signaling capabilities of different charts are analyzed The charts are permitted to run for 10 more samples after giving deviations Standard values of means and covariance matrix are used The charts for monitoring mean deviation are also analyzed when the variation is made in the process variance Similarly charts for monitoring process variance deviation are also analyzed when the deviation is made in the process means

Table-1 gives total number of problems in which process mean deviations were introduced for two and five number of variables

Table 3.1 **NUMBER OF PROBLEMS FOR MEAN CHANGES**

Number of Variables	2	5
Total No. of Problems	46 * 9	64 * 9
Total No. of Positive Problems	17 * 9	27 * 9
Total No. of Negative Problems	17 * 9	27 * 9
Total No. of Mixed Problems	12 * 9	10 * 9

Table-3 2 to Table-3 5 represents computational output (Total number of signals out of total number of problems) for various charts for different types of problems

There are three types of problems for mean deviations

- In which only positive deviations were introduced in all variables
- In which only negative deviation were introduced in all variables
- In which both positive and negative deviations were introduced

Table 3.2 **OUTPUT (MEAN CHANGES) FOR NUMBER OF VARIABLE-2**

Cor-coff.	Sample Size(n)	T2_Chart			χ^2_Chart			Cusum #1			Cusum #2		
		+	-	±	+	-	±	+	-	±	+	-	±
0 10	1	14	0	7	12	3	7	17	10	10	15	7	9
	6	16	16	12	16	15	12	17	15	12	17	15	12
	10	16	16	12	17	16	12	17	16	12	17	16	12
0 723	1	17	8	11	16	6	10	17	11	12	16	10	12
	6	17	17	12	17	17	12	17	16	12	17	17	12
	10	17	17	12	17	17	12	17	17	12	17	17	12
0 90	1	17	15	12	16	11	12	17	13	12	16	12	12
	6	17	17	12	17	17	12	17	17	12	17	17	12
	10	17	17	12	17	17	12	17	17	12	17	17	12

Table 3.3 OUTPUT (MEAN CHANGES) FOR NUMBER OF VARIABLE-2

Corr-Coff.	Smaple Sıze(n)	Expon_Chart			Z_Method			X_Bar Chart			X_Cusum Chart		
		+	-	±	+	-	±	+	-	±	+	-	±
0 10	1	17	10	10	16	10	10	15	9	9	17	10	10
	6	15	7	10	17	15	12	17	15	12	17	15	12
	10	14	12	10	17	16	12	17	16	12	17	16	12
0 723	1	17	8	12	16	7	12	16	6	11	16	6	10
	6	15	12	12	17	17	12	17	16	12	17	16	12
	10	14	12	12	16	17	12	16	17	12	17	16	12
0 90	1	17	10	12	13	13	12	17	17	12	17	7	10
	6	15	14	12	17	17	12	17	13	12	17	13	12
	10	15	15	12	17	17	12	17	16	12	17	16	12

Table 3.4 OUTPUT (MEAN CHANGES) FOR NUMBER OF VARIABLE-5

Cor-Coff.	Sample Size(n)	T2_Chart			χ^2_Chart			Cusum #1			Cusum #2		
		+	-	±	+	-	±	+	-	±	+	-	±
0 10	1	12	6	9	17	6	9	24	17	9	26	18	9
	6	23	22	10	24	23	10	26	26	10	24	24	10
	10	23	24	10	24	24	10	26	27	10	26	27	10
0 90	1	24	23	10	27	23	10	27	23	10	27	27	10
	6	23	24	10	25	25	10	24	27	10	27	27	10
	10	24	26	10	24	27	10	24	27	10	27	27	10

Table 3.5 OUTPUT (MEAN CHANGES) FOR NUMBER OF VARIABLE-5

Corr Coff.	Sample Size(n)	Z_Method			X_Bar Chart			X_Cusum Chart		
		+	-	±	+	-	±	+	-	±
0 10	1	25	19	9	17	19	8	25	19	9
	6	26	24	10	26	22	10	26	27	10
	10	26	27	10	25	26	10	26	27	10
0 90	1	20	10	10	26	18	10	26	17	10
	6	20	18	10	24	23	10	26	27	10
	10	20	20	10	26	24	10	26	24	10

Z_Method for monitoring process mean deviation, univariate Shewhart X_Bar chart and univariat X_Cusun chart also provide the idea about out-of-control and in-control variables However, it is no always true that these charts provide true result about out-of-control and in-control variables The result seem to depend on correlation coefficient, sample size and magnitude of deviation introduced in the process mean Table-6 provides the number of problems in which these charts gave true result fo different magnitude of correlation coefficient, sample size and number of variable

Table 3.6 Output (Mean changes) for True Differentiation Between Out-of-Control and In-Control Variables (Numbar of Variable = 2)

Cor-Coff.	Sample Size(n)	Z_method			X_Bar Chart			X_Cusum		
		+	-	±	+	-	±	+	-	±
0 10	1	12	4	9	9	3	7	13	5	8
	6	14	13	12	17	14	11	17	14	11
	10	14	15	12	14	15	11	17	16	12
0 723	1	12	12	12	17	3	5	16	3	5
	6	12	12	12	16	15	11	16	15	11
	10	12	12	12	17	15	12	17	16	12
0 90	1	12	12	12	17	15	12	17	4	5
	6	12	12	12	17	11	10	17	11	10
	10	12	12	12	17	15	11	17	16	11

Table 3.7 **Output (Mean changes) for True Differentiation Between Out-of-Control and In-Control Variables (Numbar of Variable = 5)**

Cor-Coff.	Sample Size	Z_method			X_Bar Chart			X_Cusum Chart		
		+	-	±	+	-	±	+	-	±
0 10	1	16	4	0	4	1	0	20	10	2
	6	15	12	7	20	13	3	22	25	8
	10	15	15	8	13	14	4	25	22	10
0 90	1	12	12	12	11	8	0	25	8	5
	6	12	12	12	16	13	4	22	22	10
	10	12	12	12	18	25	8	24	27	6

Z_Method for monitoring process variance deviation, univariate X_Range method, univariate X_Cusum and univariate S^2 control chart are used for monitoiing deviation introduced in the process variance These charts suppose to give true differentiation between out-of-control and in-control variables But for these charts also as in charts for monitoring process mean deviation, it does not hold all the time that these provide true results

Table-3 8 gives total number of problems in which process mean deviations were introduced for two and five number of variables

Table 3.8 NUMBER OF PROBLEMS FOR VARIANCE CHANGES

Number of Variables	**2**	**5**
Total No. of Problems	**22* 9**	**44 * 9**

Table-3 9 and Table-3 10 represents computational output (Total number of signals out of total number of problems) for various charts for different types of problems Only positive deviations are introduced in the process variance

Table 3.9 **OUTPUT FOR NUMBER OF VARIABLE - 2**

Cor-Coff.	**Sample Size**	**Z_method**	**X_Bar Chart**	**X_Cusum Chart**	S^2 **Chart**
0 10	1	13	22	14	
	6	18	20	21	20
	10	22	22	22	22
0 723	1	4	22	20	
	6	4	22	21	22
	10	22	21	24	22
0 90	1	4	22	20	
	6	4	22	22	22
	10	4	22	20	22

Table 3.10 **OUTPUT FOR NUMBER OF VARIABLE - 5**

Cor-Coff.	**Sample Size**	**Z_method**	**X_Bar Chart**	**X_Cusum Chart**	S^2 **Chart**
0 10	1	23	32	24	
	6	13	34	20	34
	10	18	34	20	34
mixed	1	6	32	30	
	6	6	34	13	32
	10	6	34	14	32
0 90	1	6	32	27	
	6	6	33	31	33
	10	6	34	26	34

3.1 CHI-SQUARE (χ^2) CONTROL CHART :

- This chart gives almost satisfactory result at all conditions But, it gives better result at positiv deviations from target means than negative deviations especially at n = 1 as shown in Table 3.2 an Table-3 4
- When there is both positive and negative deviation in different variables, signals improve
- For the same values of deviation, positive deviation is detected faster than negative deviation
- When sample size and correlation coefficient increases detection in case of negative deviatio improves as shown in Table -3 2 and Table-3 4
- It gives fake signal of process mean being out-of-control when deviation is introduced in the proces variance almost in all problems However there is difference in the magnitude χ^2 statistics. In cas of mean deviation, the magnitude of statistics increases continuously after deviation if it is run afte introducing deviation But, in the case of variance deviation, its magnitude increases just afte deviation After that all values do not fall out-of-control if it is run after deviation Only some of th values fall out-of-control and others remain in control So there is no continuous increment o magnitude On this basis deviation type may be recognized

3.2 T_SQUARE (T^2) CONTROL CHART :

- T^2 chart is similar to Chi-Square chart, except that estimated value of means and covariance matri from past in-control data rather than standard value of these are used
- Results are almost same as in Chi-Square chart It also gives better result at positive deviations fron target means than negative deviations especially at n = 1 as shown in Table 3 2 and Table-3 4
- When there is both positive and negative deviation in variables, improved signal is observed.
- For the same values of deviation, positive deviations are detected faster than negative deviation
- When sample size and correlation coefficient increase detection of negative deviation also improves

- As in Chi_Square control chart, it also gives fake signal of process mean being out-of-control when deviation is introduced in the process variance almost in all problems However, in case of mean deviation, the magnitude of T^2 statistics increases continuously if it is run after introducing mean deviation But, in the case of variance deviation, its magnitude increases just after deviation After that all values do not fall out-of-control if it is run after deviation Only some of the values fall out-of-control and others remain in control So there is no continuous increment of magnitude On this basis deviation type may be recognized

3.3 MULTIVARIATE CUSUM CHARTS :

3.3.1 CUMULATIVE SUM OF T AND CUSUM VECTOR SCHEME :

- Cusum Vector method gives faster signals than Cumulative Sum of T (COT) at n = 1
- As sample size n increases, COT scheme starts to give faster signal than Cusum Vector method.
- It gives fake signal of process mean being out-of-control when deviation is introduced in the process variance
- At n = 1, Cusum vector method did not give signal of process being out-of-control even after 10 runs after process deviation was introduced in more problems than the problems in which it gives such signal
- As n increases, both starts to give signal, but COT signals are faster
- At n = 1, these charts performs better than other charts However as correlation coefficient increases they perform similar to other charts as shown in Table 3 2 and Table-3 4
- It gives fake signal of process mean being out-of-control when deviation is introduced in the process variance
- These charts signal in more test problems with negative deviation than T^2 chart at n = 1. As correlation coefficient increases T2 chart signals better.

3.4 MULTIVARIATE EXPONENTIALLY MOVING-AVERAGE CHART :

- This chart performs comparable to others charts at n = 1 as shown in Table-3 3, but as n increases it signal are not as fast as of other charts
- With the increase of correlation coefficient, the signals are faster
- This chart signals in more test problems with negative deviation than T^2 chart at n = 1 As correlation coefficient increases T2 chart signals better
- It gives fake signal of process mean being out-of-control when deviation is introduced in the process variance

3.4 REGRESSION-ADJUSTMENT PROCEDURE :

3.5.1 Z-METHOD FOR MONITORING MEAN DEVIATION:

- This method is good in the sense that, separate diagnostic procedure is needed this method This chart gives clear identification of out-of-control variable or set of variables.
- But, this does not hold all the time When correlation is low i e less than 0 5, we get good differentiation between out-of-control and in-control variables As correlation increases, out-of-control variables also affect other variables
- At low correlation coefficient this chart performs similar to Shewhart X_Bar chart, but as correlation coefficient increases, deviation diffused in other variables also which does not happen in case of X_Bar chart and univariate X_Cusum chart as shown in Table-3 6 and Table-3.7 At large number of variable also, this diffusion occurs in this chart Hence at high correlation coefficient and large number of variables this chart does not perform better as X_Bar and X_Cusum performs
- It gives fake signal of process mean being out-of-control when deviation is introduced in the process variance almost in all cases Since statistics of this chart is based on cumulative sum Hence there is continuous increment of statistics even in the case of process variance deviation as in the case of process mean deviation.

3.5.2 UNIVARIATE SHEWHART X-BAR CHART :

- Separate univariate X_bar chart is run for all p variables to observe the difference in the result due to correlation coefficient
- This chart gives more clear result about which variable or set of variables are out-of-control
- It gives fake signal of process mean being out-of-control when deviation is introduced in the process variance almost in all cases

3.5.3 UNIVARIATE X-CUSUM CHART :

- Separate Univariate X_Cusum charts are also maintained for all p variables
- This chart performs satisfactory even at 0 9 correlation coefficient as shown in Table-3 6 and Table-3 7
- This chart better than Z_Method and X_Bar chart as it is clear from Table-3 6 and Table-3 7.

In the next section, we discuss the performance of charts designed to detect process variance deviation

3.6 MONITORING MULTIVARIATE PROCESS VARIANCE

3.6.0 Z-METHOD FOR MONITORING PROCESS VARIANCE

- At low correlation coefficient and number of variables, there is more number of clear differentiation between out-of-control and in-control variables, but as they increases, deviation diffused into other variables also as shown in Table-3 9 and Table-3 10. So there is less number of clear signals Other charts perform better than this chart.

3.6.1 MULTIVARIATE PROCESS VARIANCE CHART :

- It gives fake signal of process variance being out-of-control when deviation is introduced in the process mean almost in all cases
- This chart also does not identify which variable or set of variables is out-of-control It gives only overall information that process is going out-of-control
- When this chart is run for process variance deviation, it gives very good result After one run after deviation it gives out-of-control signal

3.6.2 UNIVARIATE S^2 CONTROL CHART :

- The drawback of above mentioned variance chart is described in Section-2 6 1 It's suggested to use above mentioned variance chart in conjunction of univariate S^2 control chart
- This chart gives better result than sample covariance method
- It has diagnostic capability, to identify variable or set of variables which is/are out-of-control.
- This chart perform satisfactory all correlation and sample size but it perform better al low number of variables as there is large number of clear signal of out-of-control variables as shown in Table-3.9 and Table-3 10
- When this chart is run for process mean deviation, out-of-control signals are generally not observed
- This chart performs better than Z_Method and univariate X_Cusum chart

3.6.3 UNIVARIATE X-RANGE CHART :

- Separate univariate X-Range chart is run for each of p variables
- This chart does not perform as good as S^2 control chart when process mean deviation is introduced This gives out-of-control signal when deviation is introduced in process mean
- This chart performs similar to S^2 control chart in variance deviation as shown in Table-3 9 and Table-3 10

3.7 Design of Experiment

As discussed in **Section-3.1,** all charts are permitted to run for different combinations of factors The results are tabulated With these observations a factorial design of experiment (DOE) with three factors correlation coefficient, sample-size and deviations was conducted The details of factor levels as discussed above, also shown in Table-3 1

Table 3.11 **Factors of Design of Experiment**

Factors	Levels
Correlation Coefficient	**Low, Medium and High**
Sample size	**1, 6 and 10**
Deviation	**20 Combinations of Low, Medium and High**

Initially DOE is conducted for two variable cases The output are taken in the form of +1 (if a chart signals after introducing deviation) and –1 (if a chart does not signal after introducing deviation) Each experiment is run for 10 additional samples after the deviation is introduced DOE is conducted only for main effects Thus total of 180 (3 x 3 x 20) problems were solved for different charts Table-3 2 to Table-3 10 show the summary of effects of the experimental design for different charts

3.8 Analysis of DOE

From the design of experiment, following results are obtained for different charts

3.8.1 T^2 CONTROL CHART:

From the design of experiment it can be observed that following factors are significant

- **Correlation Coefficient**
- **Sample Size**

Table 3.12 Summary of Effects for T^2 Control Chart

Effect	SS	Degr of Freedom	MS	F	p
Intercept	108 8889	1	108 8889	391 1975	0
COR_COF	10 97778	2	5 488889	19 71955	2 32E-08
SAM_SIZE	8 711111	2	4 355556	15 6479	6 41E-07
DEVIATIO	8	19	0 421053	1 512687	0 08773
Error	43 42222	156	0 278348		

Following aie the obseivations made from the results as shown ın Fıg-3 1 to Fıg-3 3

- **Wıth the increase in correlation coefficient, this chart performs better i.e. signal for out-of-control becomes faster.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement ın performance is constant.**
- **At high deviatıon, signals are faster than low deviation.**

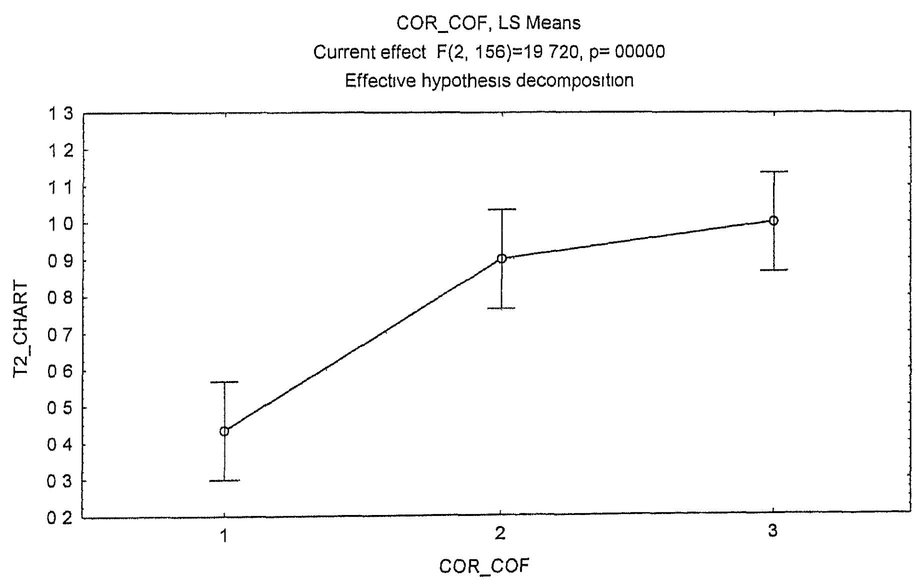

Figure 3.1 T^2 **CHART vs. CORRELATION COEFFICIENT**

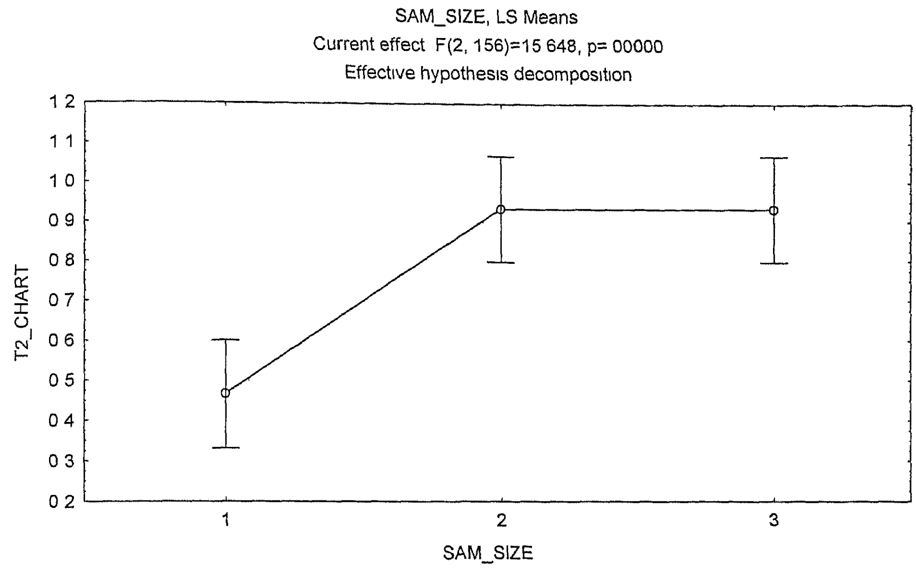

Figure 3.2 T^2 **CHART vs. SAMPLE SIZE**

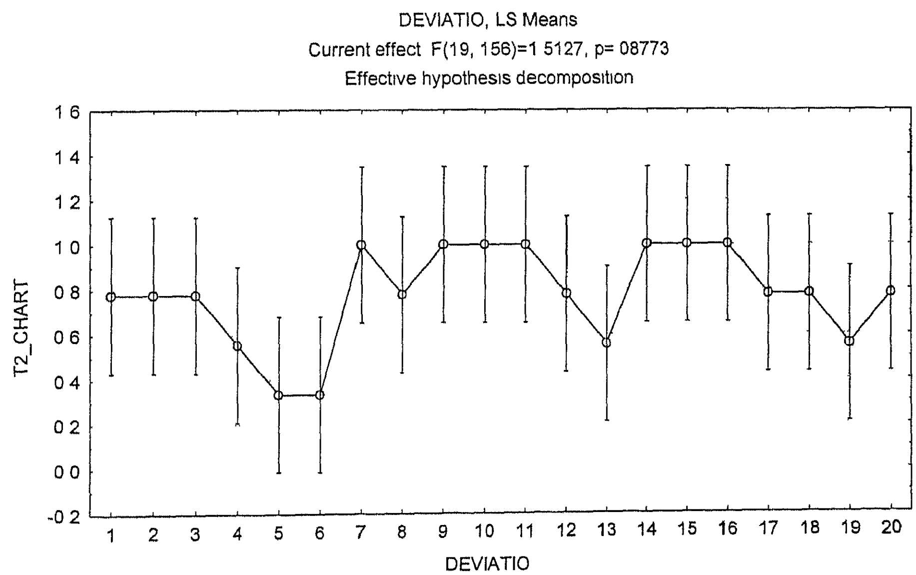

Figure 3.3 T^2 **CHART vs. DEVIATION**

3.8.2 MULTIVARIATE CUSUM CHARTS

3.8.1.1 CUSUM #1 :

Fiom the design of experiment it can be observed that following factors are significant.

- **Deviation**
- **Sample Size**

Table 3.13 Summary of Effects for Cusum #1

Effect	SS	Degr of Freedom	MS	F	p
Intercept	131 7556	1	131 7556	672 1831	0
COR_COF	0 844444	2	0 422222	2 15407	0 119449
SAM_SIZE	3 244444	2	1.622222	8 276163	0 000384
DEVIATIO	13 57778	19	0 71462	3 645808	3 45E-06
Error	30 57778	156	0 196011		

Following are the observations made fiom the iesults as shown in Fig-3 4 to Fig-3 6

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. After n = 6, performance improves slowly.**
- **At high deviation, signals are faster than low deviation.**

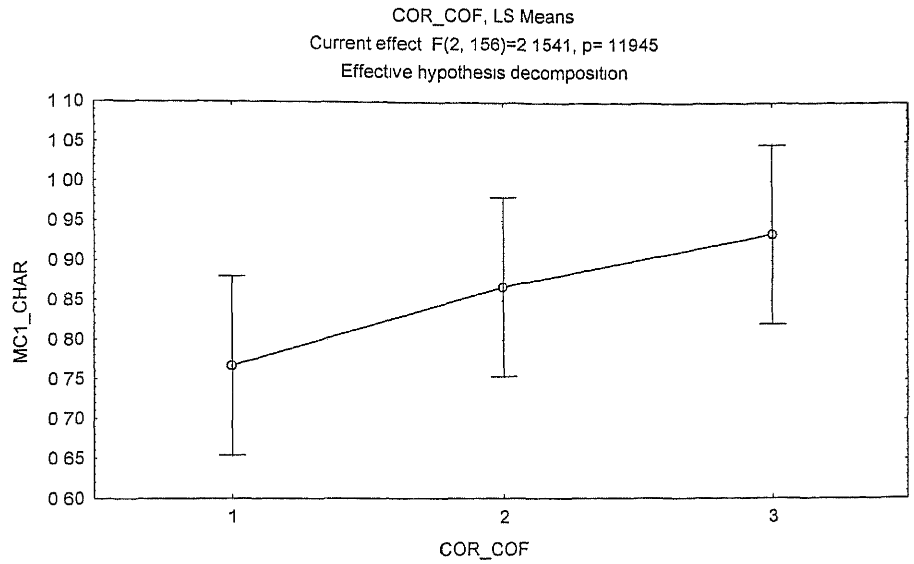

Figure 3.4 **CUSUM#1 vs. CORRELATION COEFFICIENT**

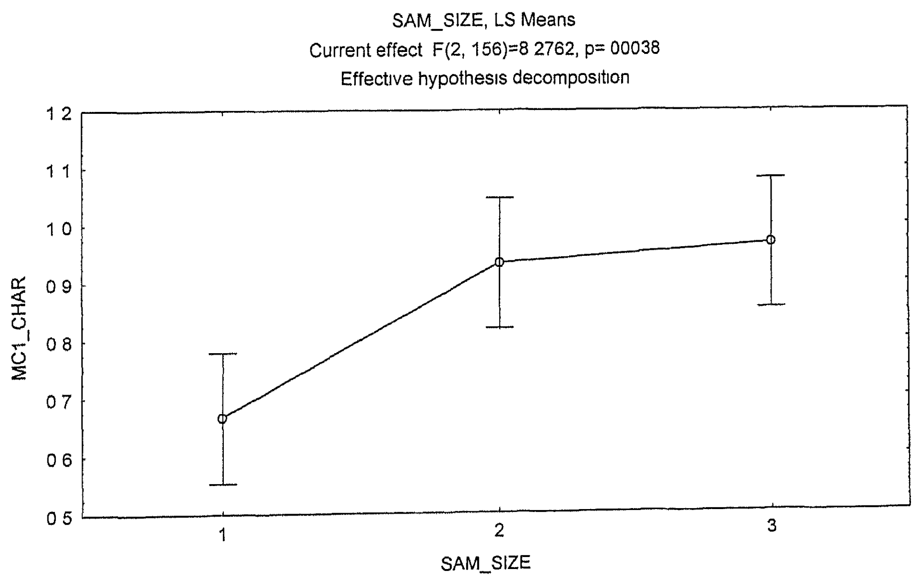

Figure 3.5 **CUSUM#1 vs. SAMPLE SIZE**

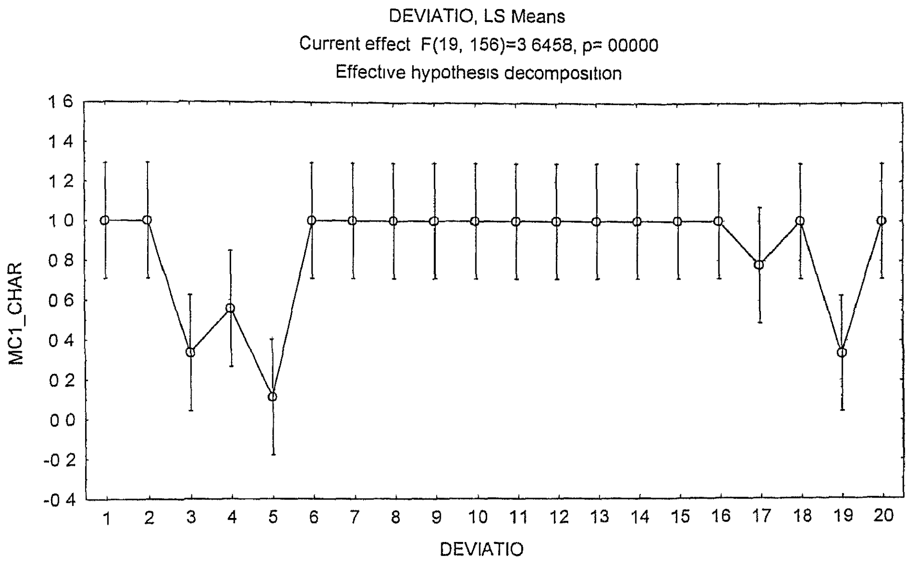

Figure 3.6 **CUSUM#1 vs. DEVIATION**

3.8.2.2 CUSUM #2:

From the design of experiment it can be observed that following factors are significant

- **Correlation**
- **Deviation**
- **Sample Size**

Table 3.14 **Summary of Effects for Cusum #2**

Effect	SS	Degr of Freedom	MS	F	p
Intercept	121 6889	1	121 6889	513 9928	0
COR_COF	4 444444	2	2 222222	9 386282	0 000142
SAM_SIZE	7 511111	2	3 755556	15 86282	5 36E-07
DEVIATION	9 422222	19	0 495906	2 094623	0 007231
Error	36 93333	156	0 236752		

Following are the observations made from the results as shown in Fig-3 7 to Fig-3 9

- **With the increase in correlation coefficient, this chart performs better.**
- **After 0.723 correlation coefficient , improvement in performance is constant.**
- **As sample size n increases, performance increases. After n = 6, improvement is**

constant.

- At high deviation, signals are faster than low deviation.

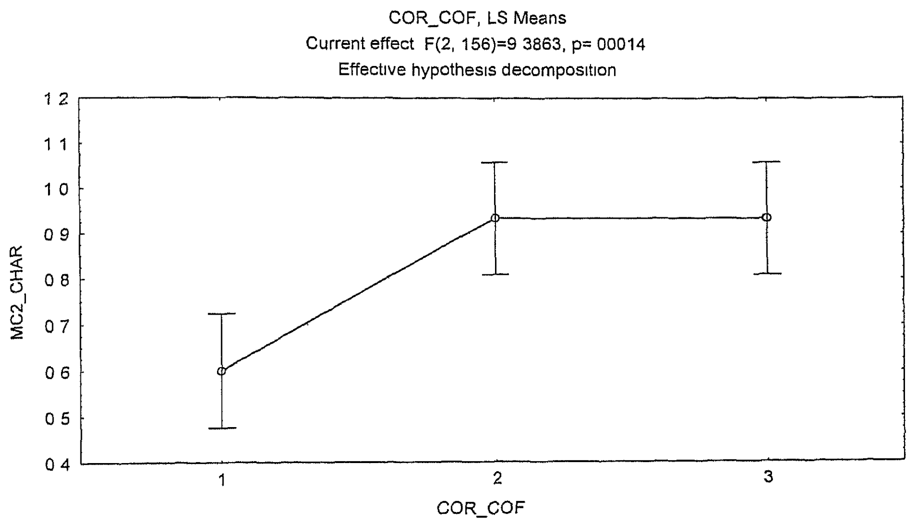

Figure 3.7 CUSUM#2 vs. CORRELATION COEFFICIENT

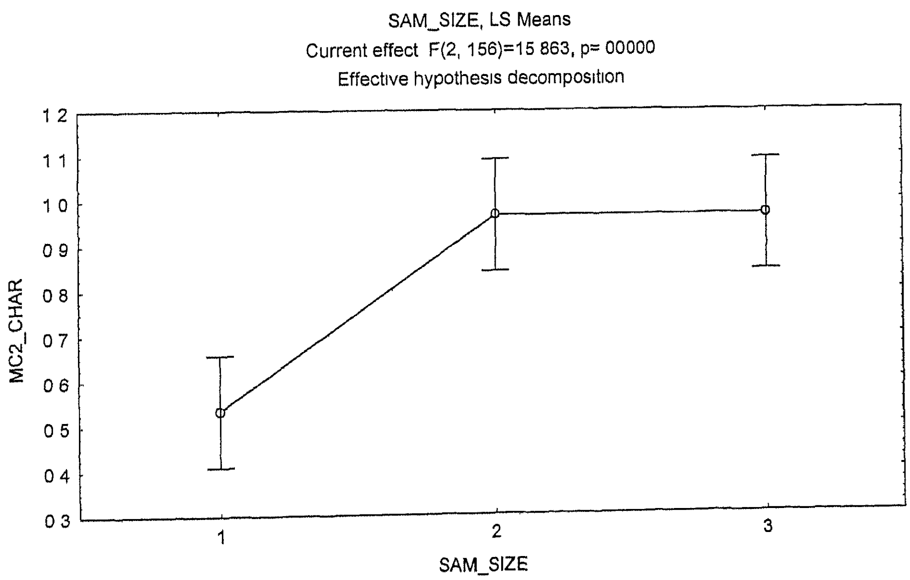

Figure 3.8 CUSUM#2 vs. SAMPLE SIZE

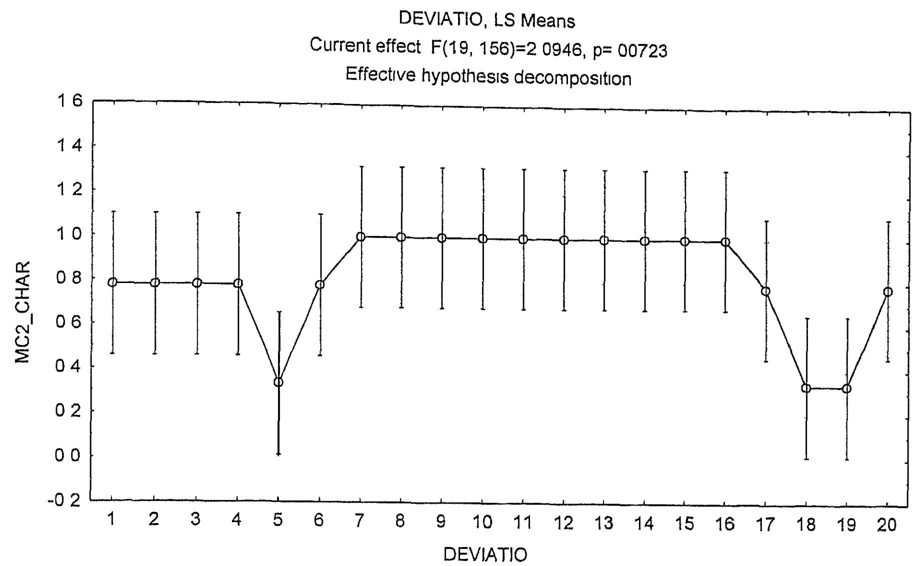

Figure 3.9 **CUSUM#2 vs. DEVIATION**

3.8.3 SHEWHART X_BAR CHART :

From the design of experiment it can be observed that following factors are significant

- **Deviation**
- **Sample Size**

Table 3.15 **Summary of Effects for X_Bar Chart**

Effect	SS	Degr of Freedom	MS	F	p
Intercept	105 8	1	105 8	437 4064	0
COR_COF	1 2	2	0 6	2 480565	0 086992
SAM_SIZE	3 733333	2	1 866667	7 717314	0 000637
DEVIATIO	31 53333	19	1 659649	6 861447	6 31E-13
Error	37 73333	156	0 24188		

Following are the observations made from the results as shown in Fig-3 10 to Fig- 3 12

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, improvement in performance is slower.**
- **At high deviation, signals are faster than low deviation**

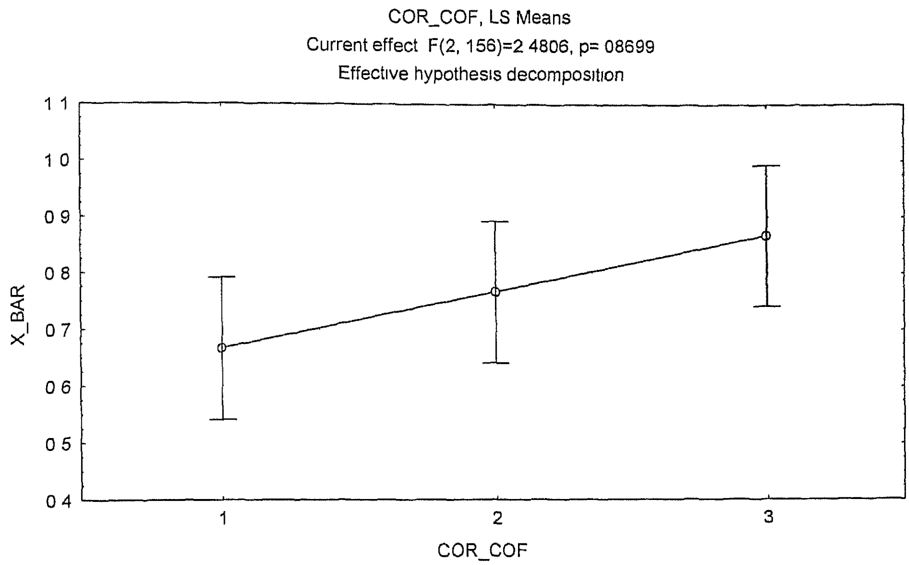

Figure 3.10 X-BAR CHART vs. CORRELATION COEFFICIENT

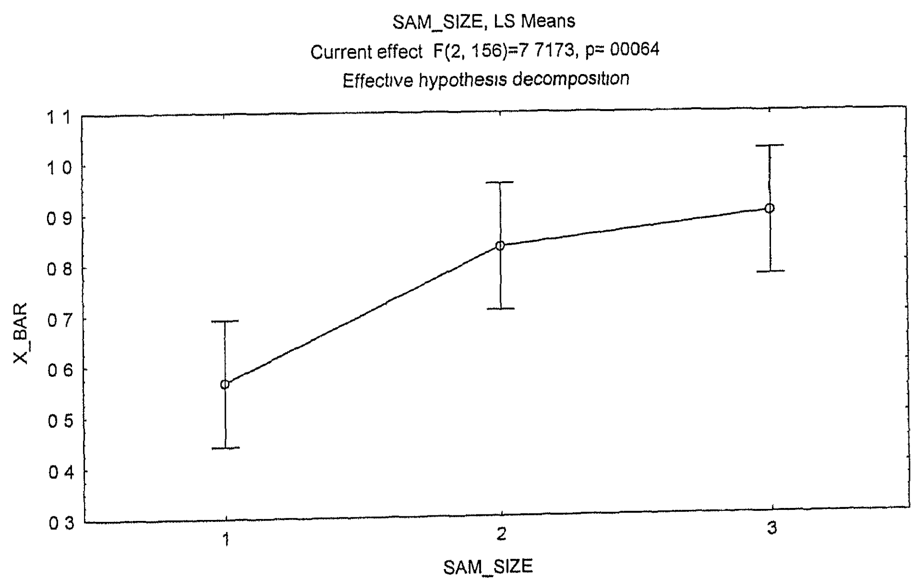

Figure 3.11 X-BAR CHART vs. SAMPLE SIZE

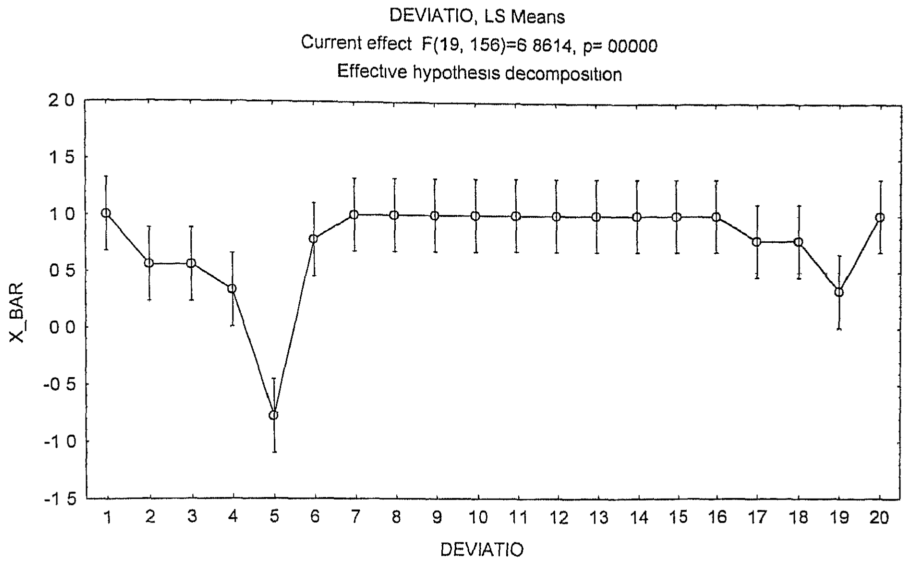

Figure 3.12 X_BAR CHART vs. DEVIATION

3.8.4 UNIVARIATE X_CUSUM CHART :

From the design of expeiiment it can be observed that following factors are significant.

- **Deviation**
- **Sample Size**

Table 3.16 Summary of Effects for X_Cusum Chart

Effect	SS	Degr of Freedom	MS	F	p
Intercept	93 88889	1	93 88889	433 0486	0
COR_COF	0 311111	2	0 155556	0 717477	0.489585
SAM_SIZE	7 644444	2	3 822222	17 62943	1 25E-07
DEVIATION	44 33333	19	2 333333	10 76216	0
Error	33 82222	156	0 216809		

Following are the observations made from the results as shown in Fig-3 13 to Fig-3.15

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, improvement in performance is slower.**
- **At high deviation, signals are faster than low deviation**

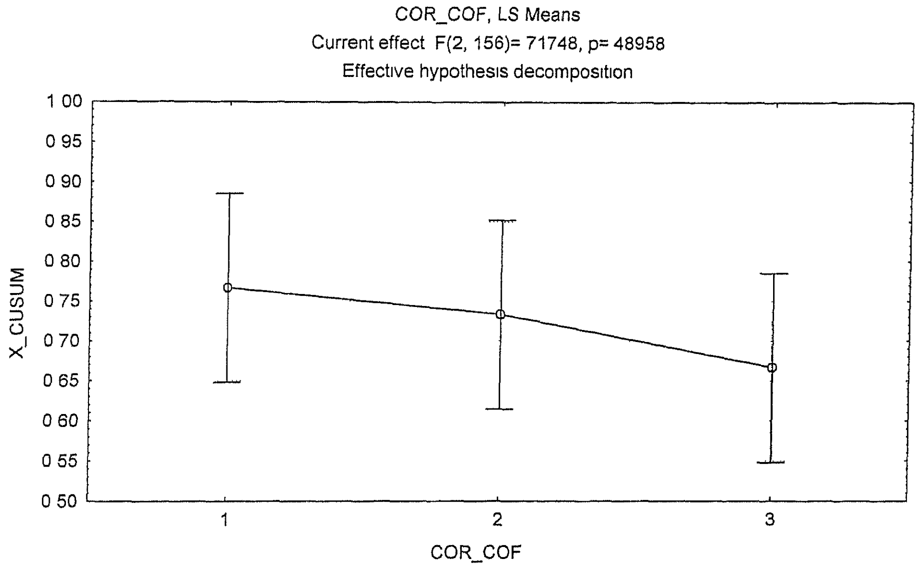

Figure 3.13 X_CUSUM vs. CORRELATION COEFFICIENT

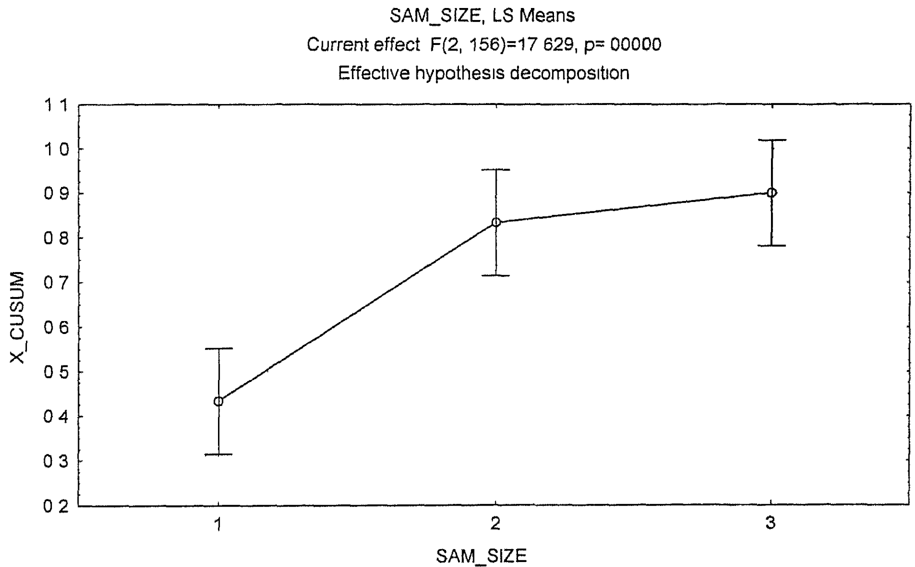

Figure 3.14 X_CUSUM vs. SAMPLE SIZE

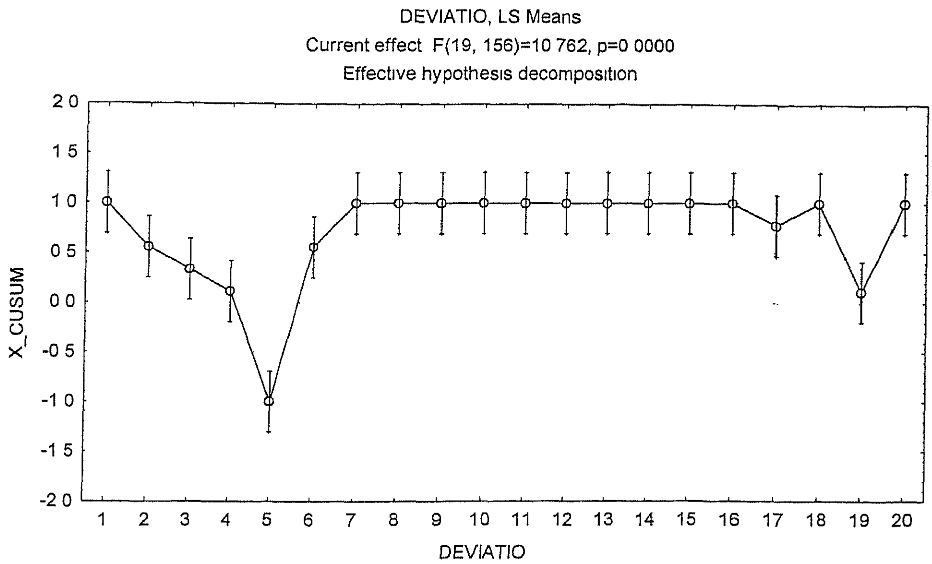

Figure 3.15 **X_CUSUM vs. DEVIATION**

3.8.5 Z_METHOD :

From the design of experiment it can be observed that following factors are significant·

- **Deviation**
- **Sample Size**

Table 3.17 **Summary of Effects for Z_Method**

Effect	SS	Degr of Freedom	MS	F	p
Intercept	125	1	125	543.0074	0
COR_COF	0 933333	2	0 466667	2.027228	0 135155
SAM_SIZE	4 933333	2	2 466667	10 71535	4 36E-05
DEVIATION	13 22222	19	0 695906	3 023059	7 94E-05
Error	35 91111	156	0 230199		

Following are the observations made from the results as shown in Fig-3 16 to Fig-3.18

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, improvement in performance starts to decrease.**

- **At high deviation, signals are faster than low deviation**

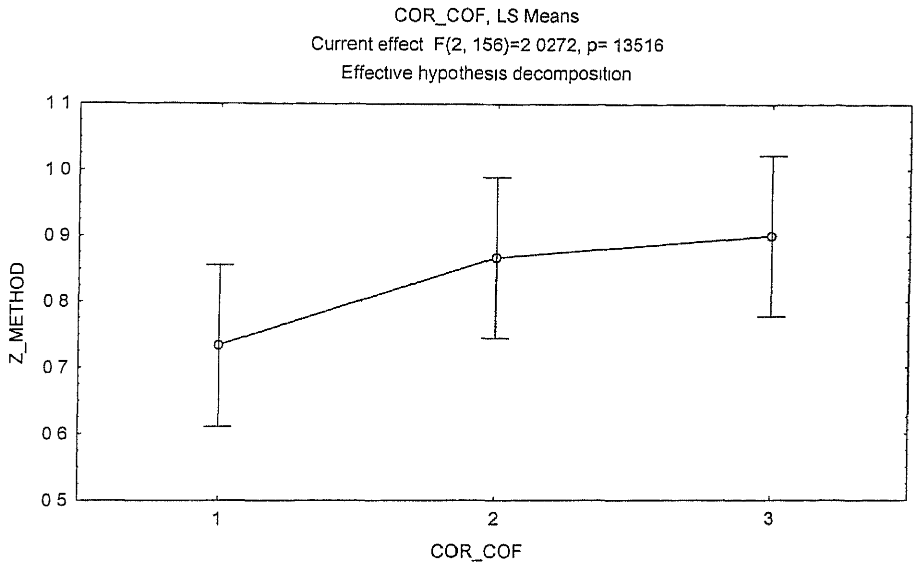

Figure 3.16 Z_METHOD vs. CORRELATION COEFFICIENT

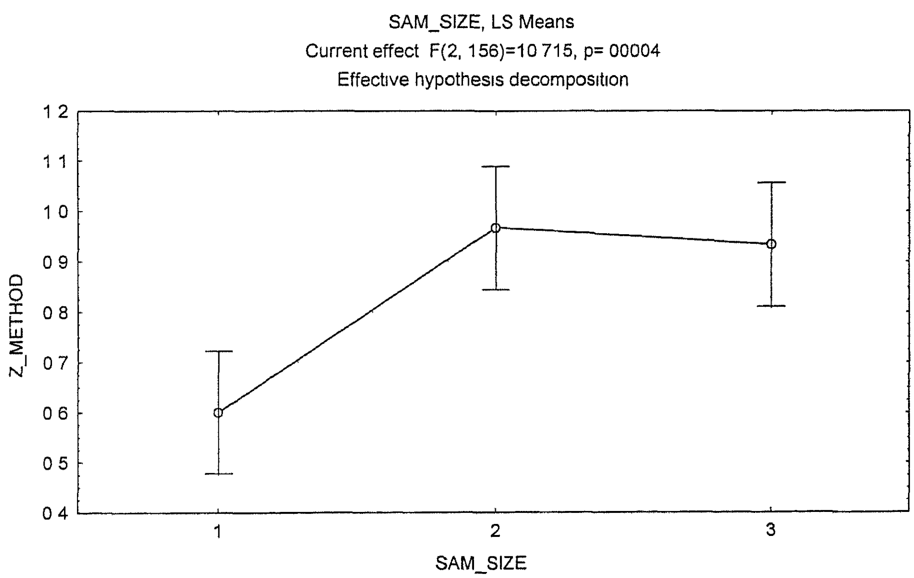

Figure 3.17 Z_METHOD vs. SAMPLE SIZE

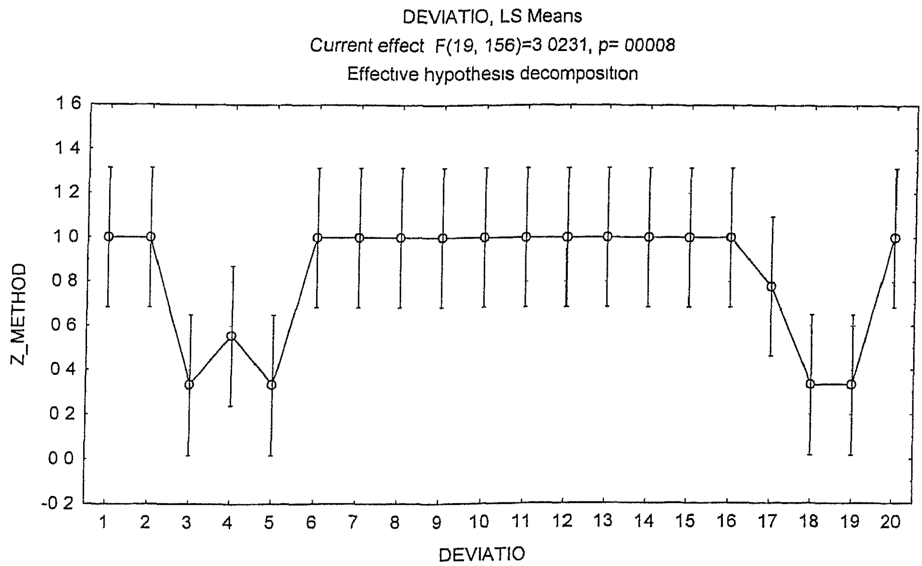

Figure 3.18 **Z_METHOD vs. DEVIATION**

3.8.6 EXPON_CHART :

From the design of experiment it can be observed that following factors are significant

- **Deviation**

Table 3.18 **Summary of Effects for Expon_Chart**

Effect	SS	Degr of Freedom	MS	F	p
Intercept	11 8642	1	11 8642	29 81204	7 26E-07
COR_COF	1 876543	2	0 938272	2 357664	0 102334
SAM_SIZE	0 395062	2	0 197531	0 49635	0 610937
DEVIATIO	39 80247	8	4 975309	12 50182	7 65E-11
Error	27 06173	68	0 397967		

Following are the observations made from the results as shown in Figure 3 19 to 3 21

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, improvement starts to decrease and after n = 6 it becomes constant.**
- **At high deviation, signals are faster than low deviation**

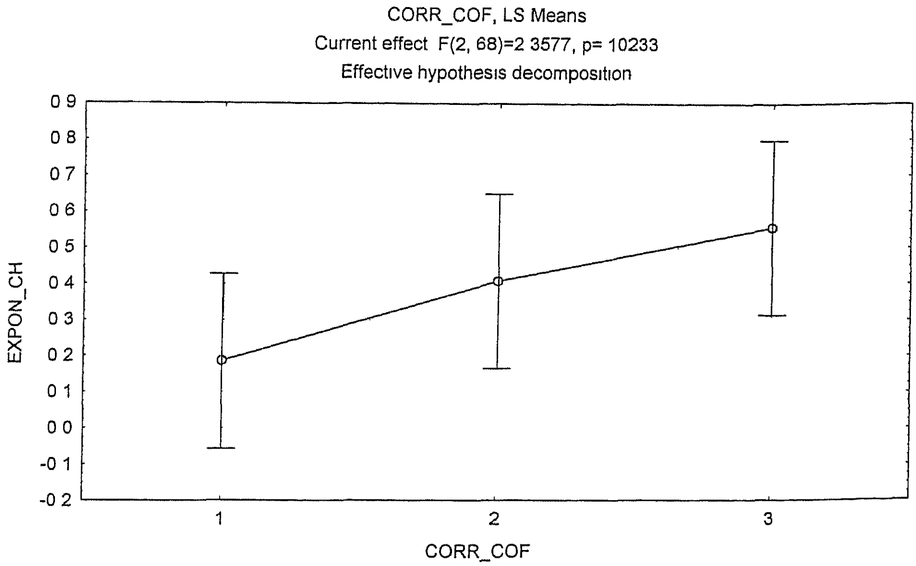

Figure 3.19 EXPON_CHART vs. CORRELATION COEFFICIENT

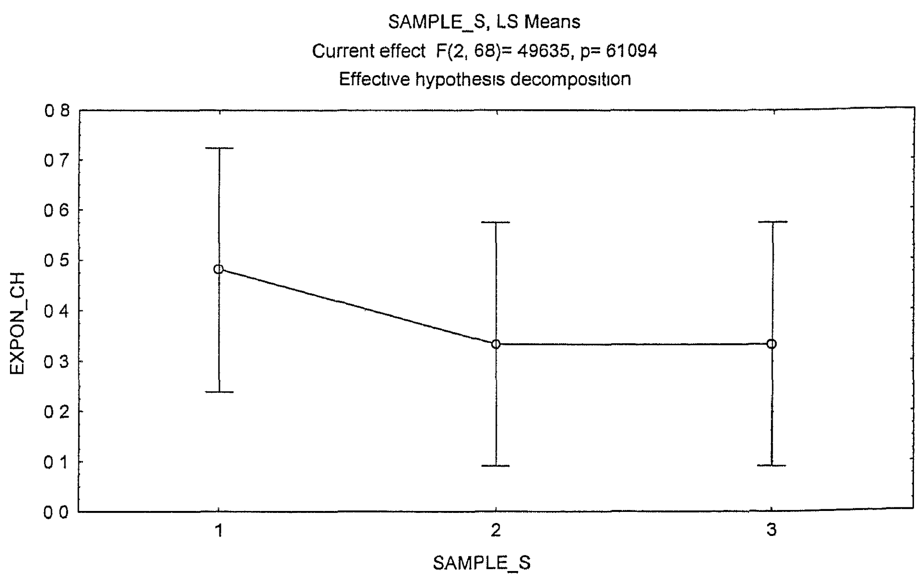

Figure 3.20 EXPON_CHART vs. SAMPLE SIZE

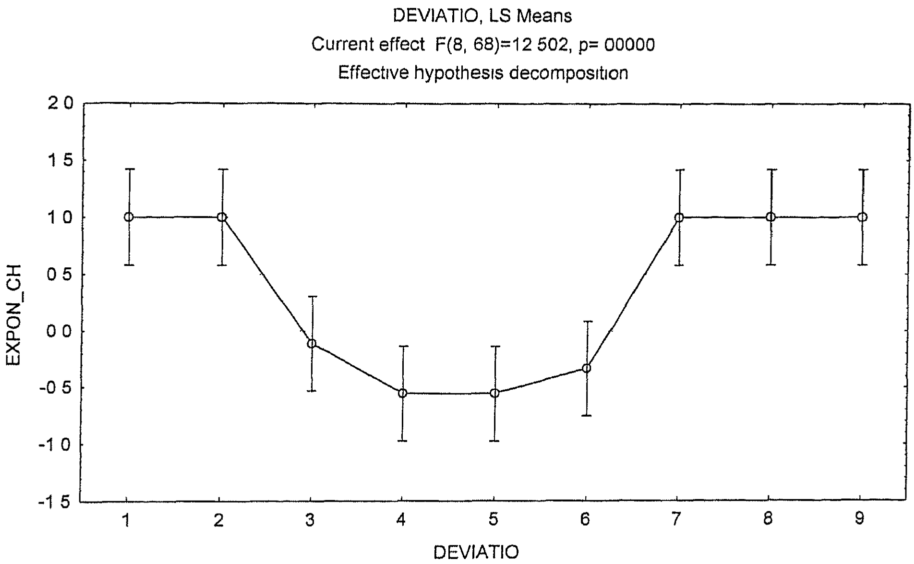

Figure 3.21 **EXPON_CHART vs. DEVIATION**

3.9 Design of Experiment with Detection (Nondetection) as Performance Measure

From the previous DOE, it is observed that deviation in fact can be clustered into 3 or 4 levels. So in this set of experiment, we select the detection of signal as two-performance level For this purpose if signal is detection 10 samples after the deviation is introduced, an output value of +1 (Detection) is assigned otherwise output value of –1 (Nondetection) is assigned Also, this DOE considered all effects i e interactions effects also The DOE is replicated 5 times and analyzed for six charts Thus total of 180 problems (3 * 3 * 4 * 5) were solved for each chart.

Table 3.19 Factors of Design of Experiment

Factors	Levels
Correlation Coefficient	Low, Medium and High
Sample Size	1, 6 and 10
Deviation	4 Combinations of Low, Medium and High

Levels of Deviation:

- **Low** $< \pm 0.5$
- **Medium** $> \pm 0.5$ **and** $< \pm 1.0$
- **High** $> \pm 1.0$
- **High and Low Combinations of Deviation**

3.10 Analysis of DOE

From the design of experiment, following results are obtained for different charts

3.10.1 T^2 CONTROL CHART :

From the design of experiment it can be observed that following factors are significant

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**
- **Correlation Coefficient and Sample Size**
- **Correlation Coefficient and Deviation**

Table 3.20 Summary of Effects for T^2 Control Chart

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	3 622222	144	0.255556	14 17391	2 4E-06
2	2	4 355556	144	0 255556	17 04348	2 27E-07
3	3	1 481481	144	0 255556	5 797101	0 000906
12	4	1 088889	144	0 255556	4 26087	0 00274
13	6	1 103704	144	0 255556	4 318841	0 000489
23	6	0 148148	144	0 255556	0 57971	0 746052
123	12	0 17037	144	0 255556	0 666667	0 781

Following are the observations made from the results as shown in Fig-3 22 to Fig- 3.26

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement in performance is constant.**
- **In the figure 3.25, third level of deviation factor is combination of high positive and high negative. T2 chart does not signal properly at negative high deviation, that's why output is less than fourth level at which is T2 chart performs good.**
- **In two-way interaction effect between sample size and correlation coefficient, at particular sample size, as correlation coefficient increases, performance improves. At n = 1, number of signals are more due to increase in correlation coefficient as compared to n > 1.**
- **In two-way interaction effect between deviation and correlation coefficient, at particular deviation, as correlation coefficient increases, performance improves.**

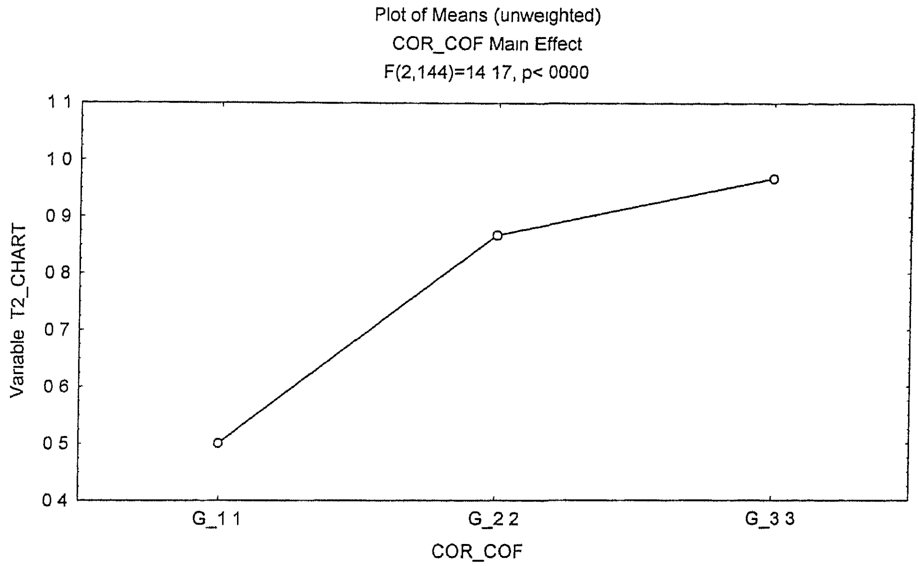

Figure 3.22 T^2 **CHART vs. CORRELATION COEFFICIENT**

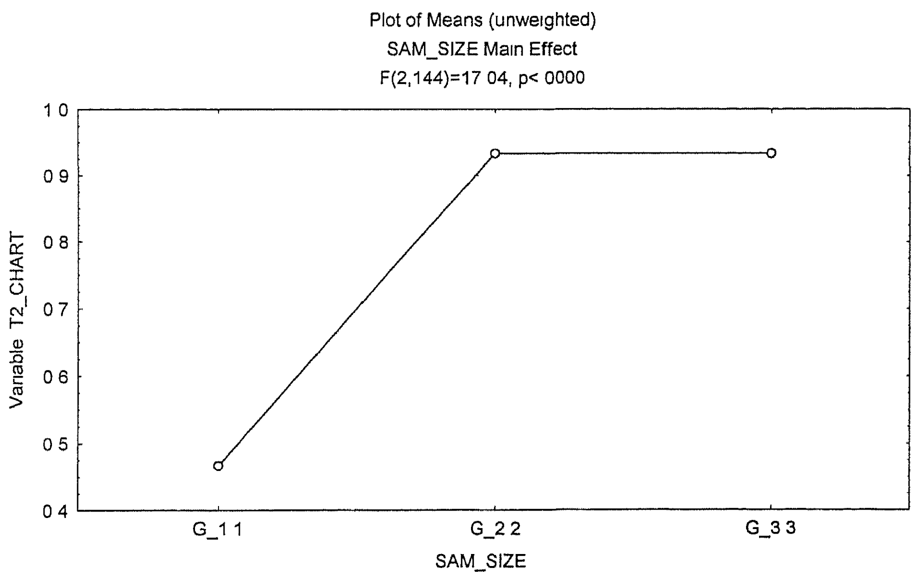

Figure 3.23 T^2 **CHART vs. SAMPLE SIZE**

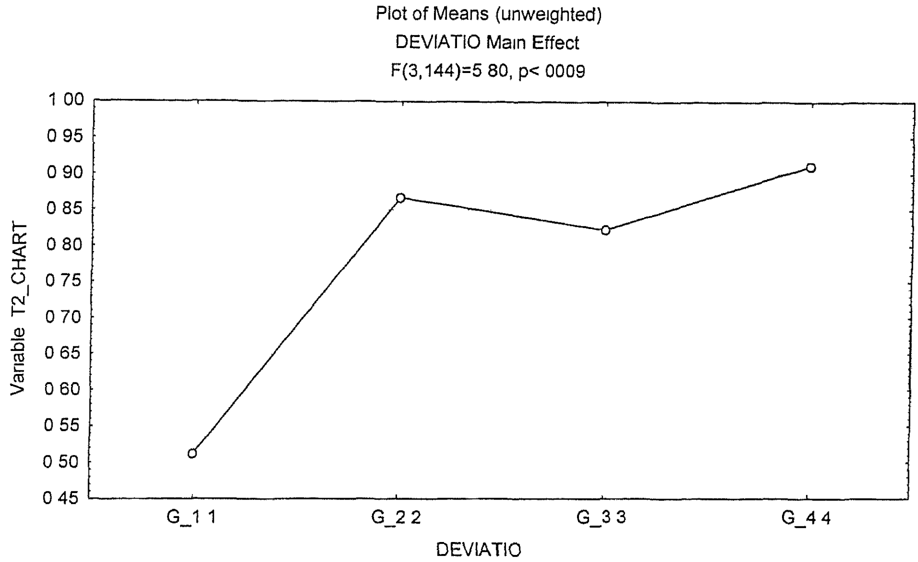

Figure 3.24 T^2 **CHART vs. DEVIATION**

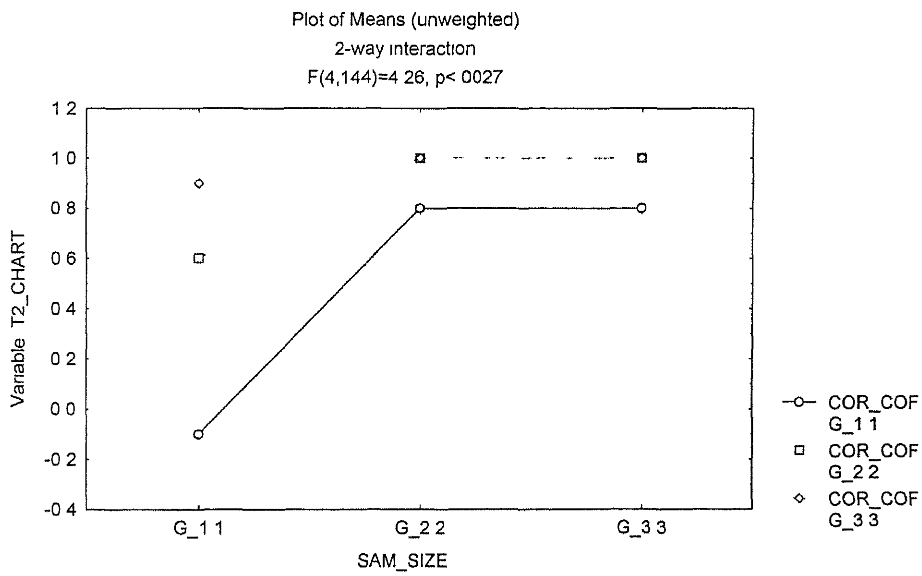

Figure 3.25 T^2 **CHART vs. CORRELATION COEFFICIENT AND SAMPLE SIZE**

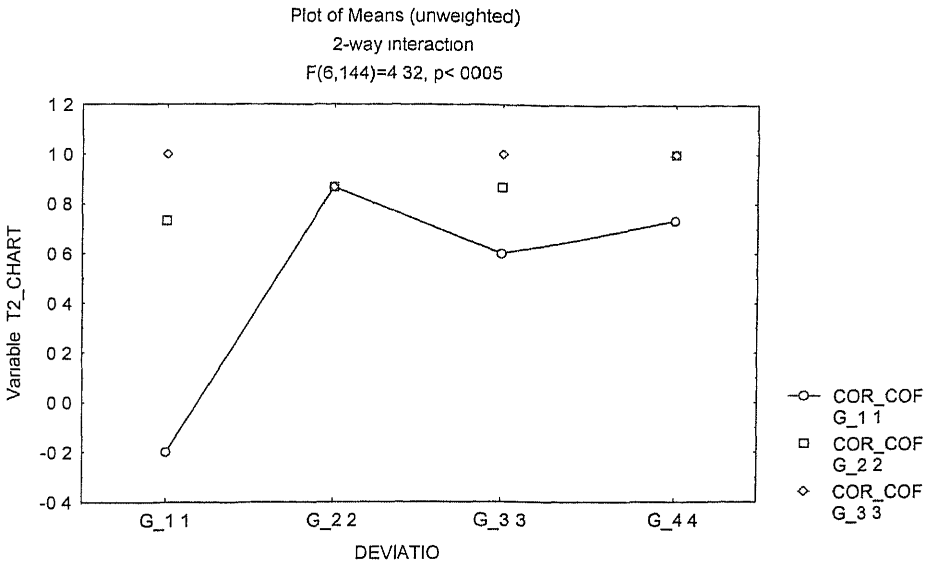

Figure 3.26 T^2 **CHART vs. CORRELATION COEFFICIENT AND DEVIATION**

3.10.2 MULTIVARIATE CUSUM CHARTS

3.10.2.1 CUSUM #1 :

From the design of experiment it can be observed that following factors are significant

- **Sample Size**
- **Deviation**
- **Sample Size and Deviation**

Table 3.21 Summary of Effects for Cusum #1

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	0 288889	144	0.233333	1 238095	0 293003
2	2	1 622222	144	0 233333	6 952381	0 001311
3	3	1 622222	144	0 233333	6 952381	0 000211
12	4	0 055556	144	0 233333	0 238095	0 916421
13	6	0 288889	144	0 233333	1 238095	0 290286
23	6	0 555556	144	0 233333	2 380952	0 031861
123	12	0 055556	144	0 233333	0 238095	0 996041

Following are the observations made from the results as shown in Fig-3 27 to Fig-3 30

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, performance improvement is slow.**
- **With increase in the deviation, charts signals are fast.**
- **In the 2-way interaction effect between sample size and deviation, at low deviation as sample size increases signals are becoming faster, but as deviation increases increment of sample size does not much effect.**

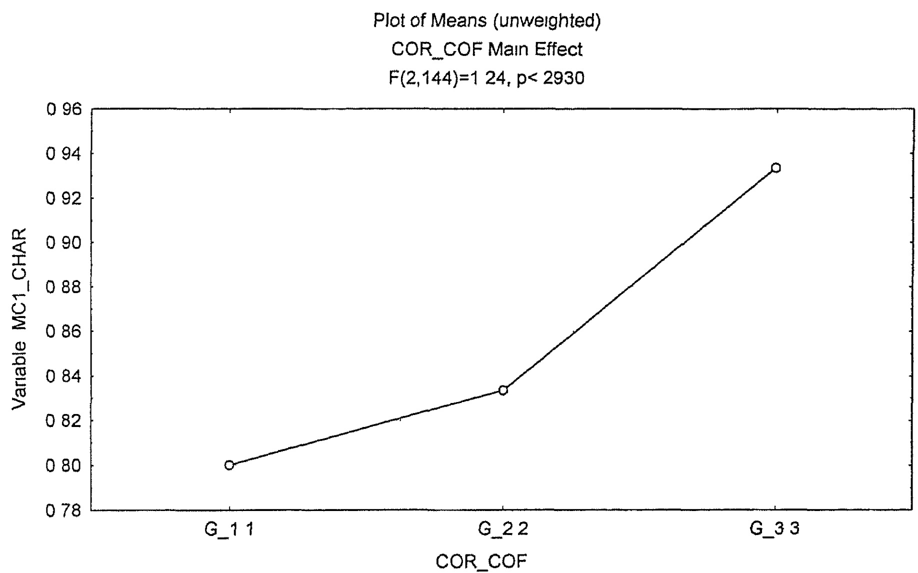

Figure 3.27 CUSUM #1 vs. CORRELATION COEFFICIENT

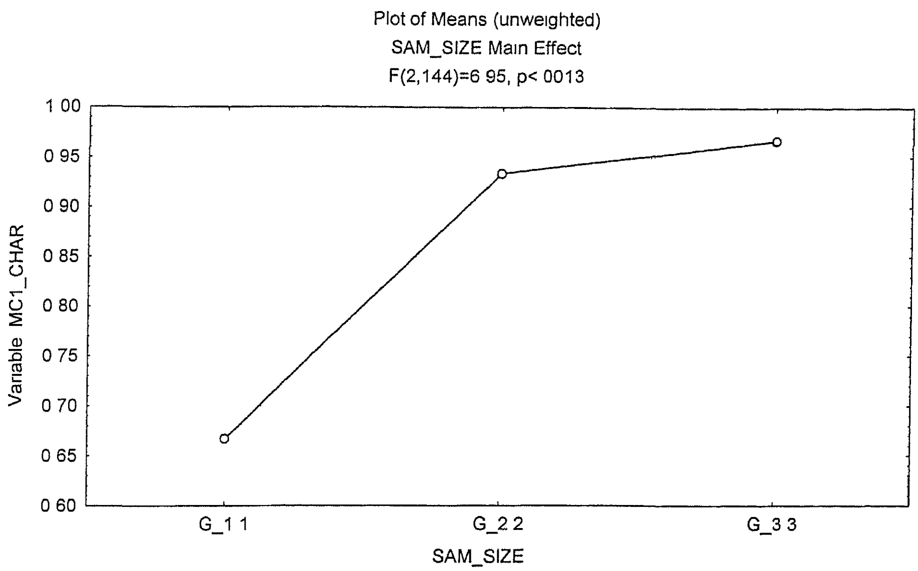

Figure 3.28 **CUSUM #1 vs. SAMPLE SIZE**

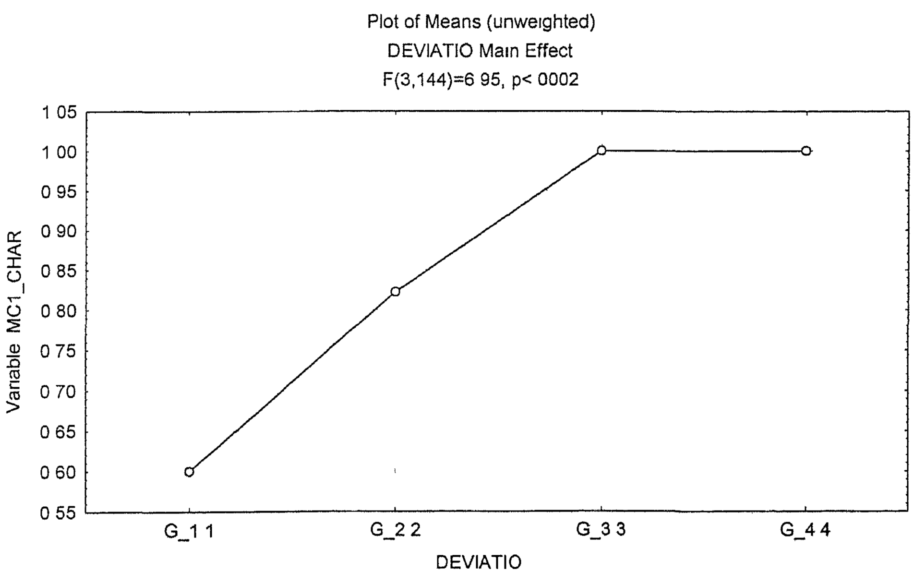

Figure 3.29 **CUSUM #1 vs DEVIATION**

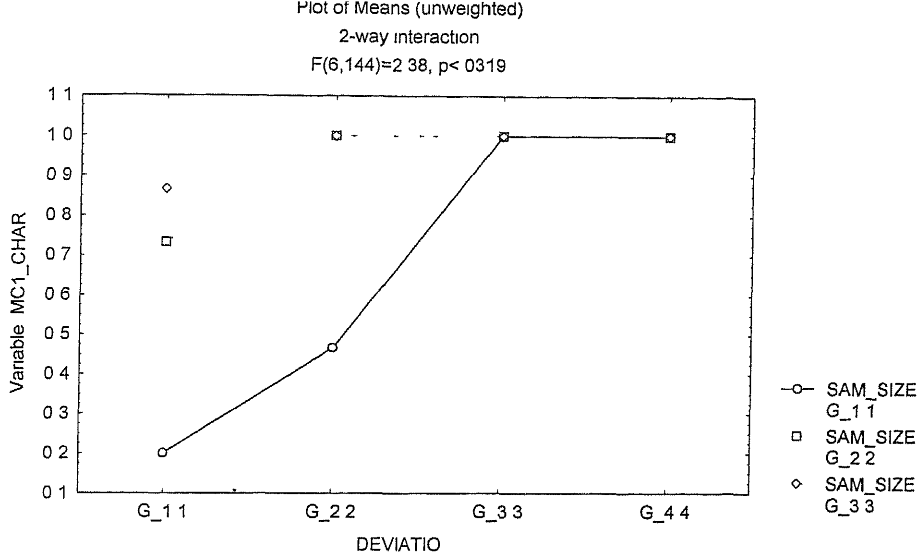

Figure 3.30 CUSUM #1 vs SAMPLE SIZE AND DEVIATION

3.10.2.2 CUSUM #2

From the design of experiment it can be observed that following factors are significant:

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**
- **Sample Size and Deviation**
- **Correlation Coefficient and Deviation**

Table 3.22 Summary of Effects for Cusum #2

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	0 8	144	0 2	4	0.020387
2	2	3 2	144	0 2	16	5 31E-07
3	3	2 214815	144	0 2	11 07407	1 38E-06
12	4	0 2	144	0 2	1	0 409714
13	6	0 503704	144	0 2	2 518518	0 023908
23	6	1 125926	144	0.2	5 62963	2 82E-05
123	12	0 081481	144	0 2	0 407407	0.958878

Followmg are the observatıons made from the results as shown ın Fıg-3 31 to Fig-3 35

- **Wıth the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, performance improvement is slow.**
- **Wıth ıncrease in the deviation, charts'signals are fast.**
- **In two-way interaction effects between correlation coefficient and deviation, correlation coefficient has more effect than deviation. All correlation coefficient'levels have the same effect. As both increases, they have same effect as they coincide.**
- **In two-way interaction effects between sample size and deviation, sample size has more effect than deviation. All sample size'levels have the same effect. As both increases, they have same effect as they coincide.**

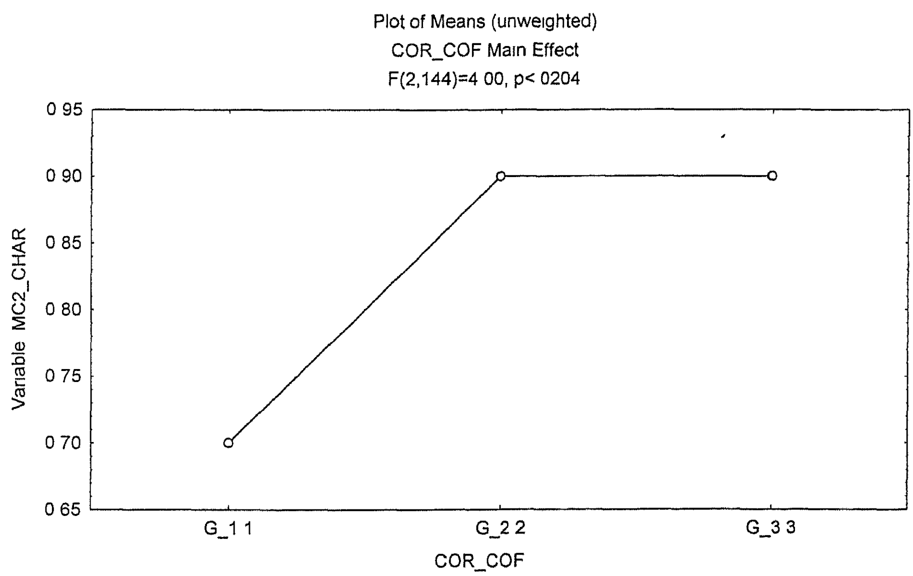

Figure 3.31 CUSUM #2 vs. CORRELATION COEFFICIENT

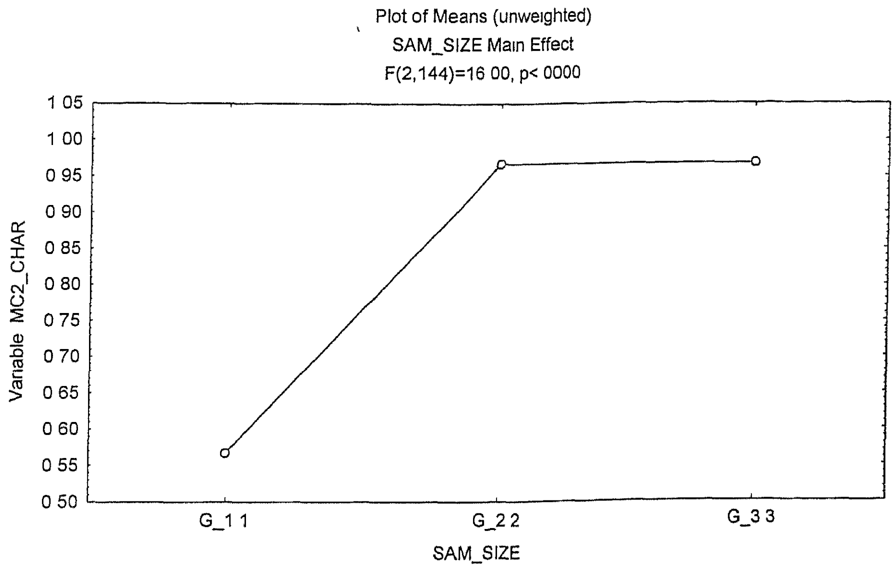

Figure 3.32 **CUSUM #2 vs. SAMPLE SIZE**

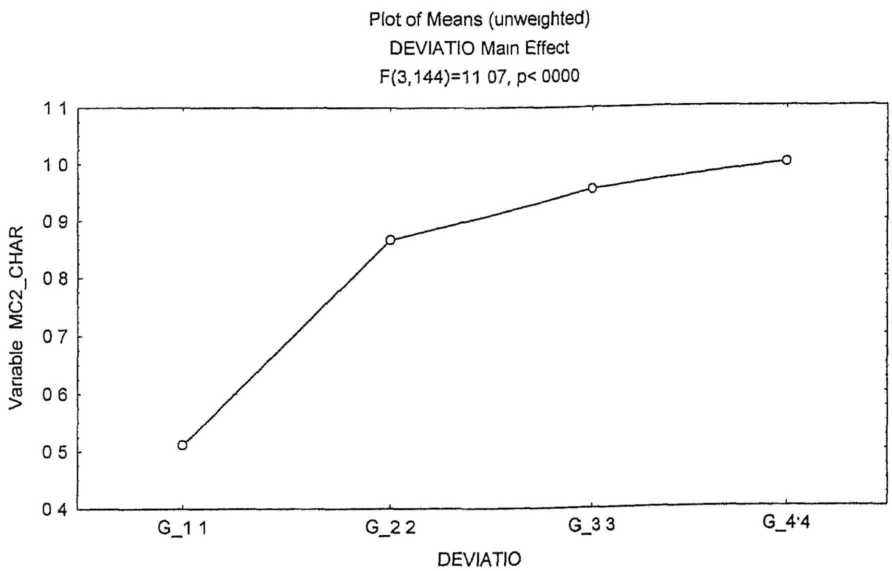

Figure 3.33 **CUSUM#2 vs. DEVIATION**

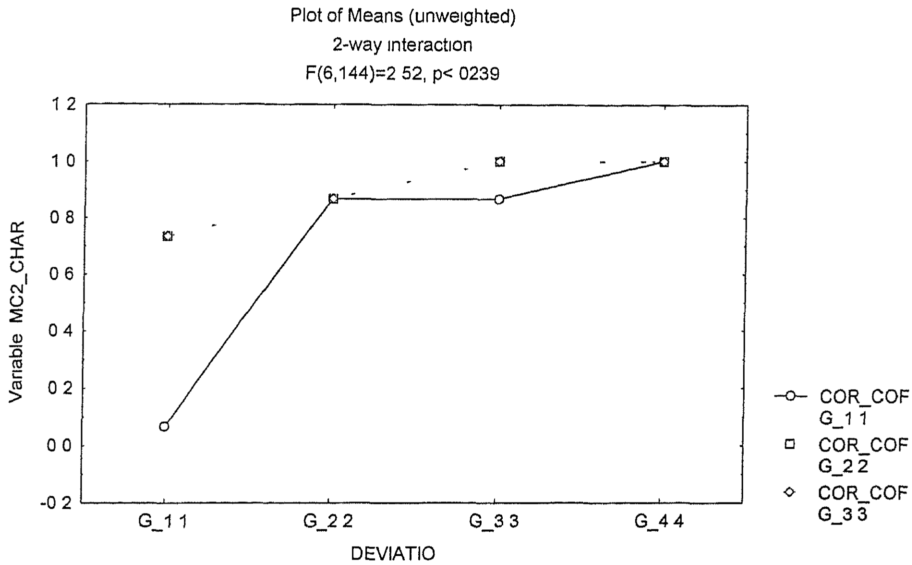

Figure 3.34 CUSUM#2 vs. CORRELATION COEFFICIENT AND DEVIATION

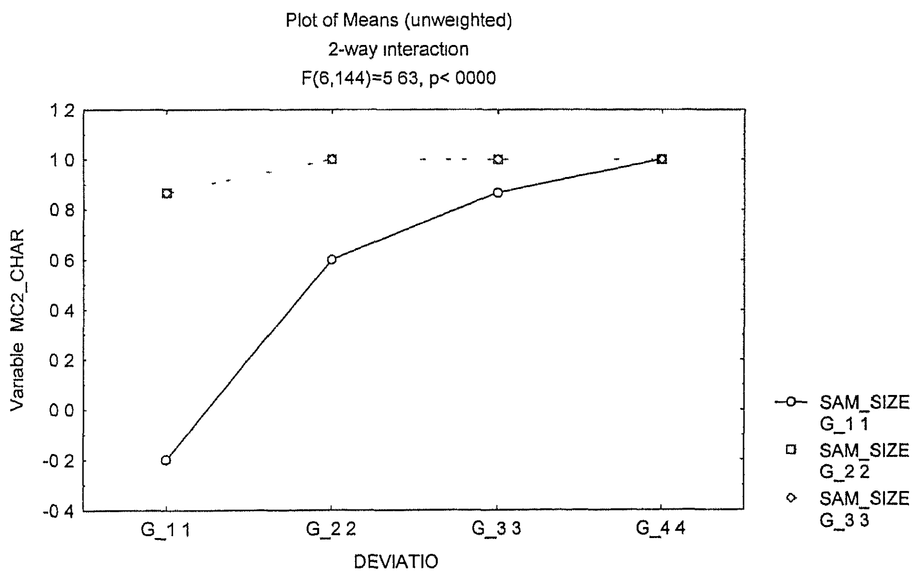

Figure 3.35 CUSUM#2 vs. SAMPLE SIZE AND DEVIATION

3.10.3 Z_METHOD :

From the design of experiment it can be observed that following factors are significant

- **Sample Size**
- **Deviation**
- **Sample Size and Deviation**

Table 3.23 Summary of Effect for Z_Method

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	0 288889	144	0 233333	1 238095	0 293003
2	2	2 955555	144	0 233333	12 66667	8 56E-06
3	3	2 844445	144	0 233333	12 19048	3 73E-07
12	4	0 055556	144	0 233333	0 238095	0 916421
13	6	0 288889	144	0 233333	1 238095	0 290286
23	6	1 177778	144	0 233333	5 047619	9 98E-05
123	12	0 055556	144	0 233333	0 238095	0 996041

Following are the observations made from the results as shown in Fig-3 36 to Fig-3 39

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance deteriorate.**
- **With increase in the deviation, charts'signals are fast.**
- **In two-way interaction effects between sample size and deviation, sample size has more effect than deviation. All sample size'levels have the same effect. As both increases, they have same effect as they coincide.**

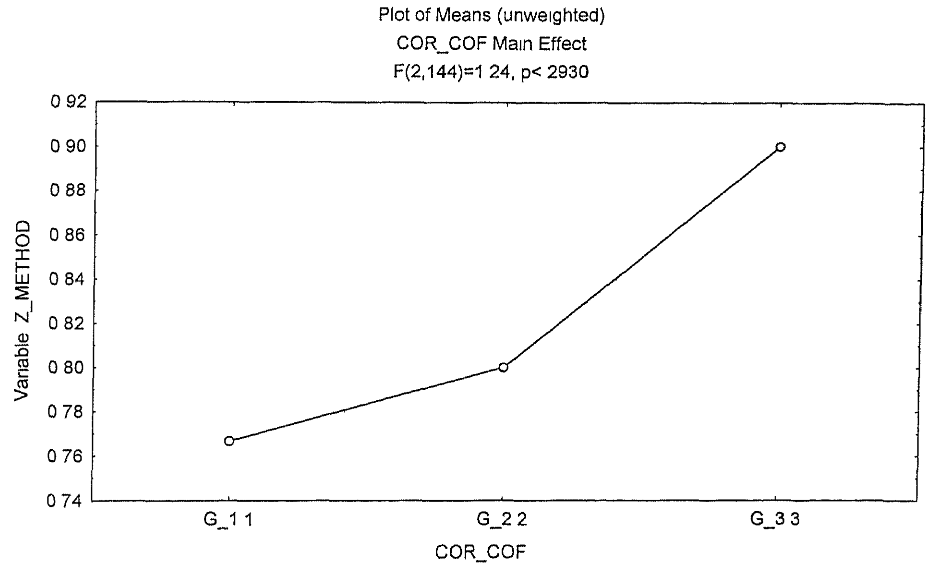

Figure 3.36 Z_METHOD vs. CORRELATION COEFFICIENT

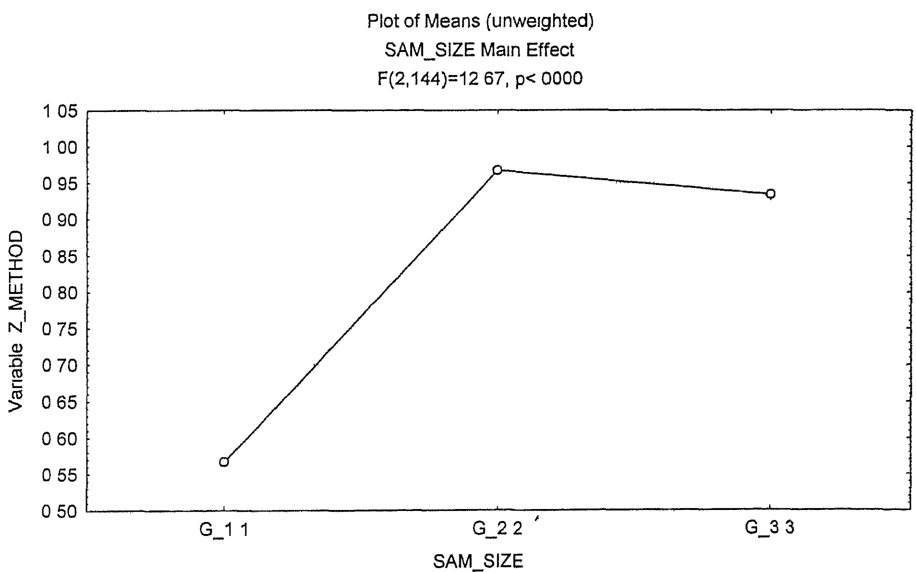

Figure 3.37 Z_METHOD vs. SAMPLE SIZE

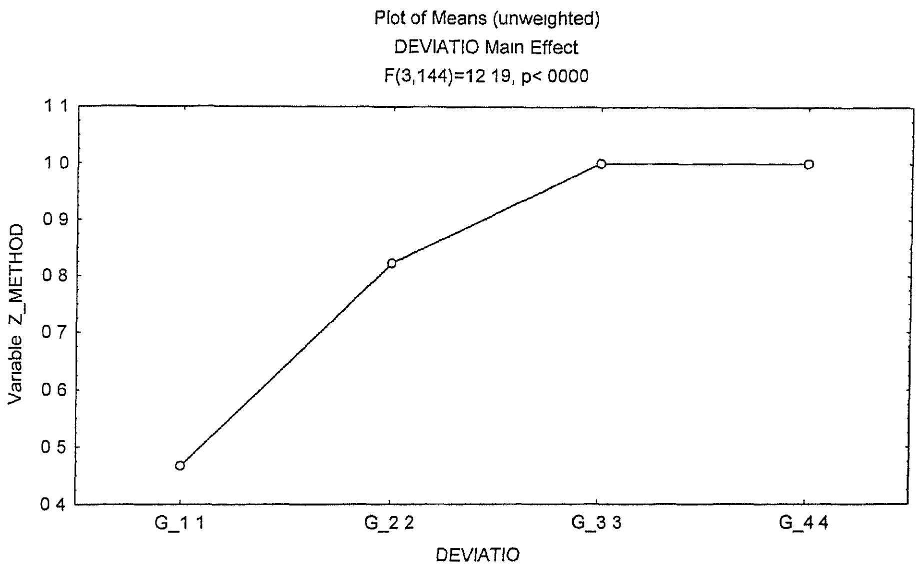

Figure 3.38 **Z_METHOD vs. DEVIATION**

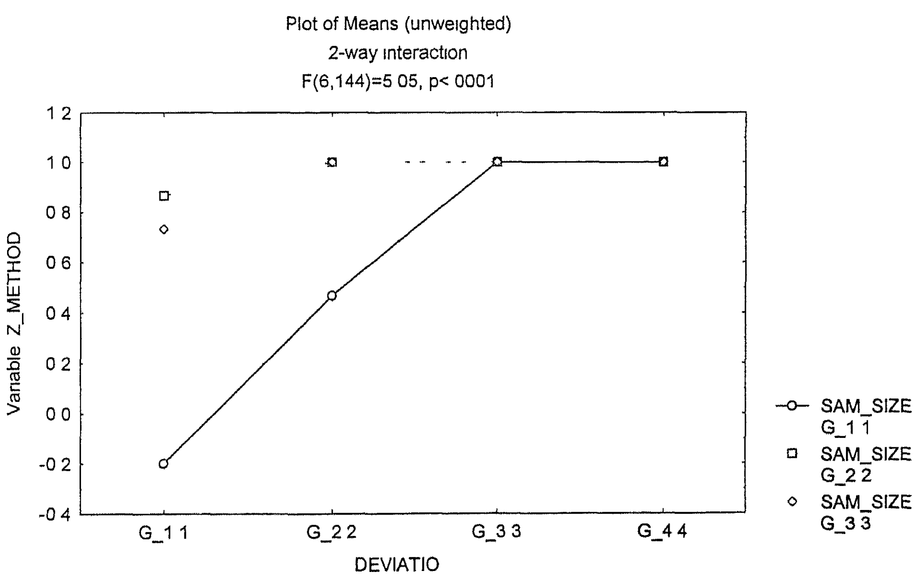

Figure 3.39 **Z_METHOD vs. DEVIATION AND SAMPLE SIZE**

3.10.4 SHEWHART X_BAR CHART :

From the desıgn of experıment ıt can be observed that followıng factors are sıgnıficant

- **Sample Size**
- **Deviation**
- **Correlatıon Coefficient and Sample Size**
- **Correlatıon Coefficient, Sample Size and Deviation**

Table 3.24 Summary of Effects for X_Bar Chart

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	0 266667	144	0 233333	1 142857	0 321782
2	2	0 866667	144	0 233333	3 714286	0 026738
3	3	4 533333	144	0 233333	19 42857	1 24E-10
12	4	0 933333	144	0 233333	4	0 004162
13	6	0 266667	144	0 233333	1 142857	0 340595
23	6	0 333333	144	0 233333	1 428571	0 207569
123	12	0 666667	144	0 233333	2 857143	0 001488

Followıng are the observatıons made from the results as shown in Fıg-3.40 to Fıg-3 44

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases.**
- **With increase in the deviation, charts'signals are fast.**
- **In two-way interaction effect between correlation coefficient and sample size, at low values both factor have same effect but as correlation coefficient increases, it has less effect than sample size.**
- **In 3-way interaction effect among correlation coefficient, sample size and deviation, at high values all have same effect.**

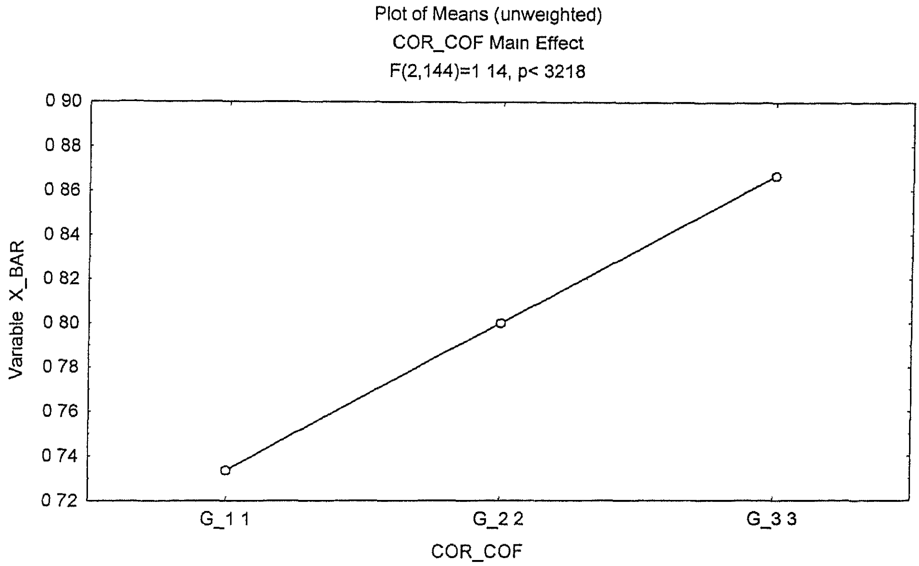

Figure 3.40 X_BAR CHART vs. CORRELATION COEFFICIENT

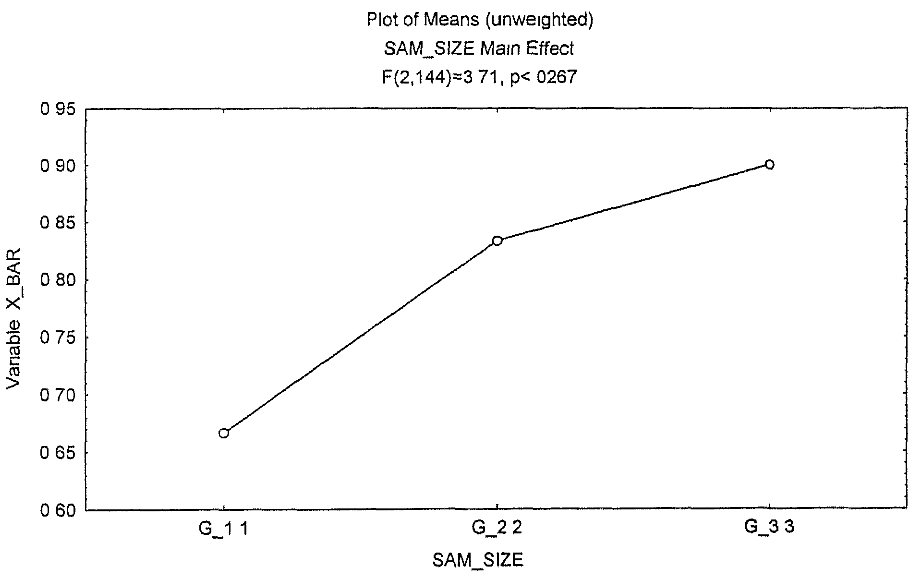

Figure 3.41 X_BAR CHART vs. SAMPLE SIZE

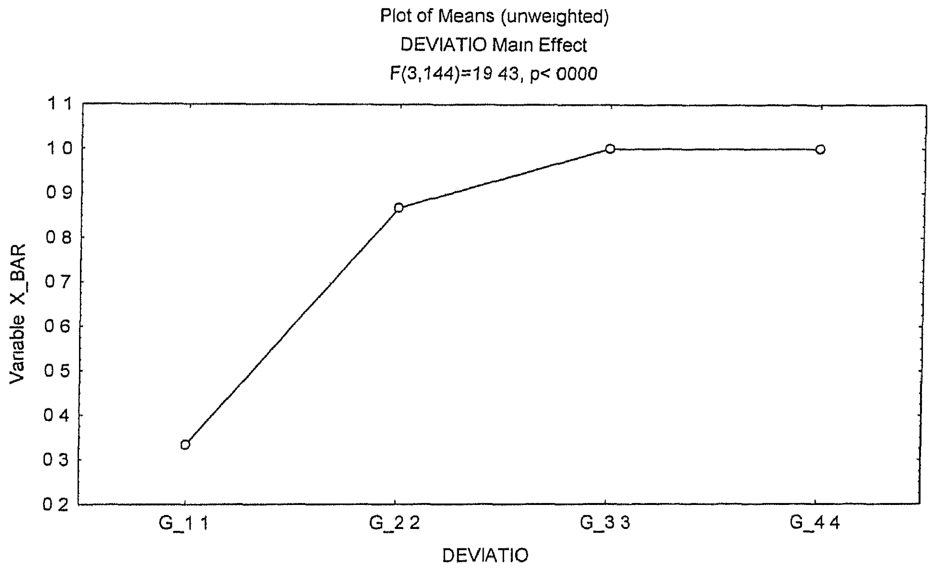

Figure 3.42 **X_BAR CHART vs. DEVIATION**

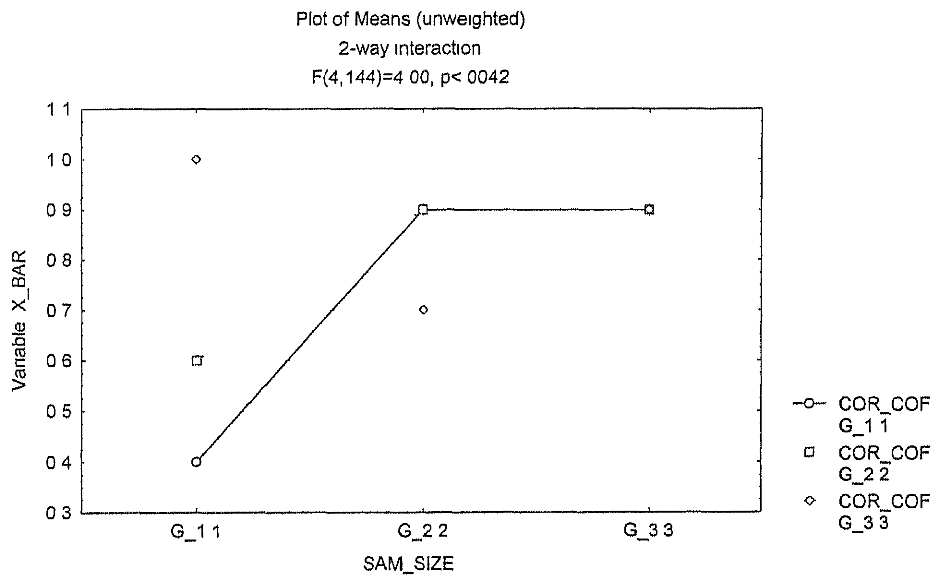

Figure 3.43 **X_BAR CHART vs. CORRELATION COEFFICIENT AND SAMPLE SIZE**

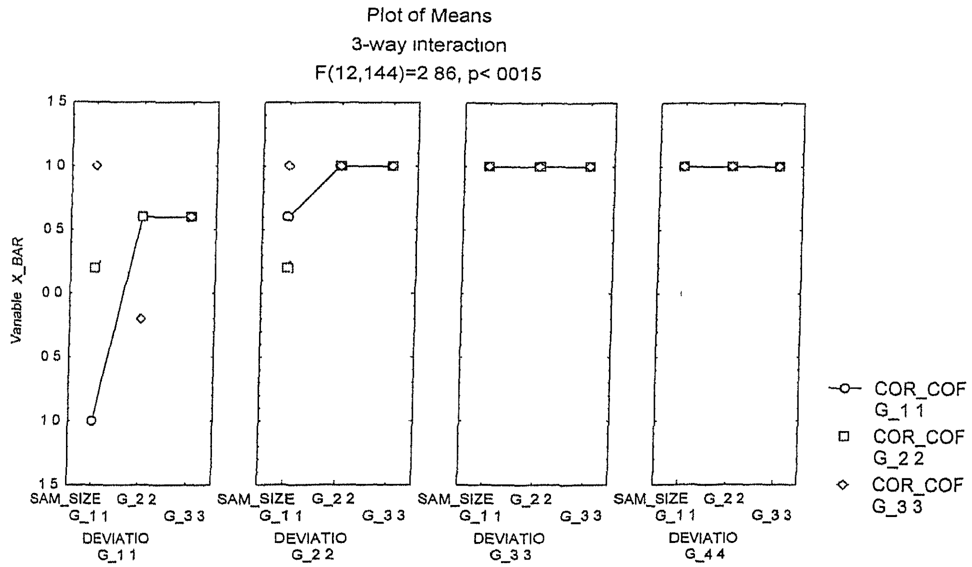

Figure 3.44 X_BAR CHART vs. CORRELATION COEFFICIENT, DEVIATION AND SAMPLE SIZE

3.10.5 UNIVARIATE X_CUSUM CHART :

From the design of experiment it can be observed that following factors are significant

- **Sample Size**
- **Deviation**
- **Sample Size and Deviation**

Table 3.25 Summary of Effects for X_Cusum

Effects	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	0 266667	144	0 311111	0 857143	0 426526
2	2	3 266667	144	0.311111	10 5	5 54E-05
3	3	7 170371	144	0 311111	23 04762	3.04E-12
12	4	0 133333	144	0 311111	0 428571	0 787819
13	6	0 148148	144	0 311111	0 47619	0 825196
23	6	1 192593	144	0 311111	3.833333	0 001413
123	12	0.103704	144	0 311111	0 333333	0 981929

Followıng are the observatıons made from the results as shown ın Fıg-3 45 to Fig-3.48

- **With the increase in correlation coefficient, this chart performs worse.**
- **As sample size n increases, performance also increases.**
- **Wıth increase in the deviation, charts'signals are fast.**
- **In two-way interaction effects between sample size and deviation, sample size has more effect than deviation. As both factor increases, they have same effect as they coincide.**

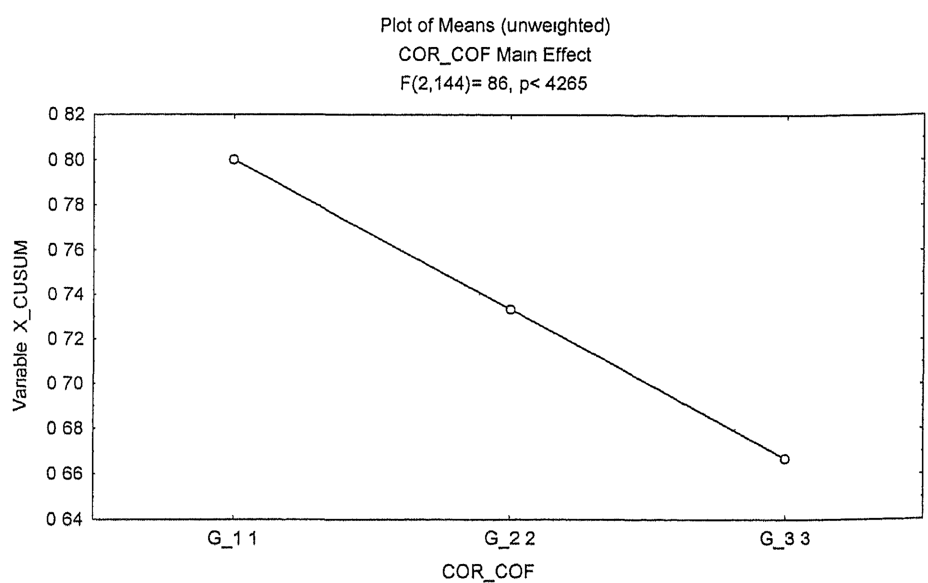

Figure 3.45 X_CUSUM vs. CORRELATION COEFFICIENT

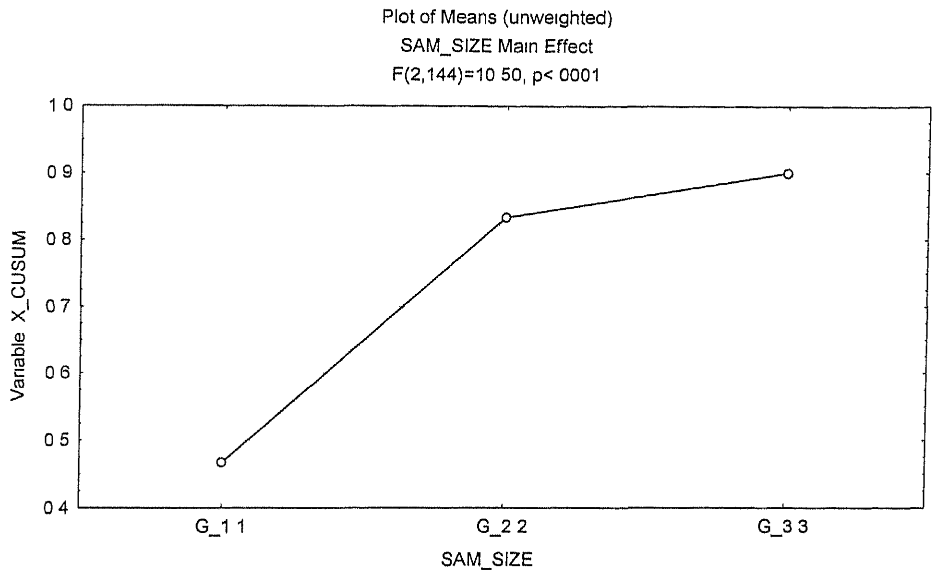

Figure 3.46 **X_CUSUM vs. SAMPLE SIZE**

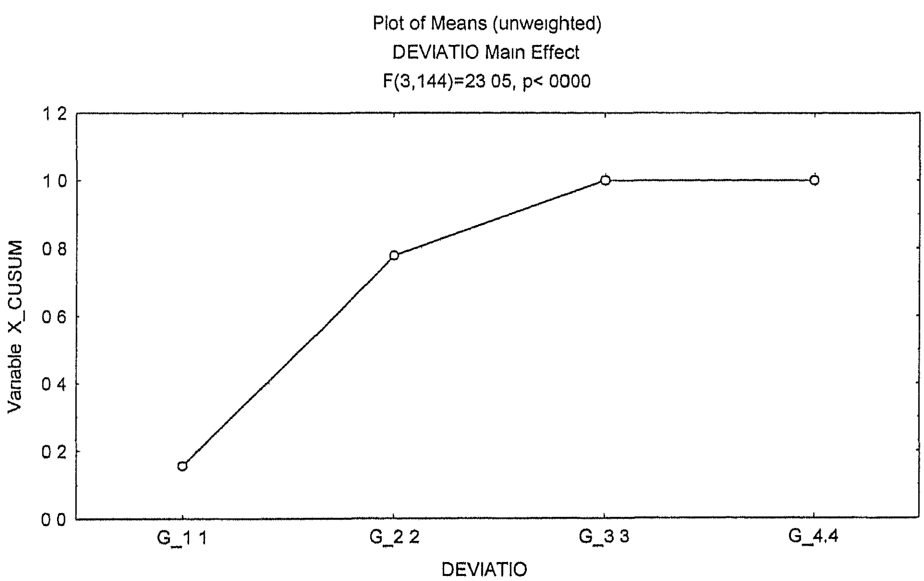

Figure 4.47 **X_CUSUM vs. DEVIATION**

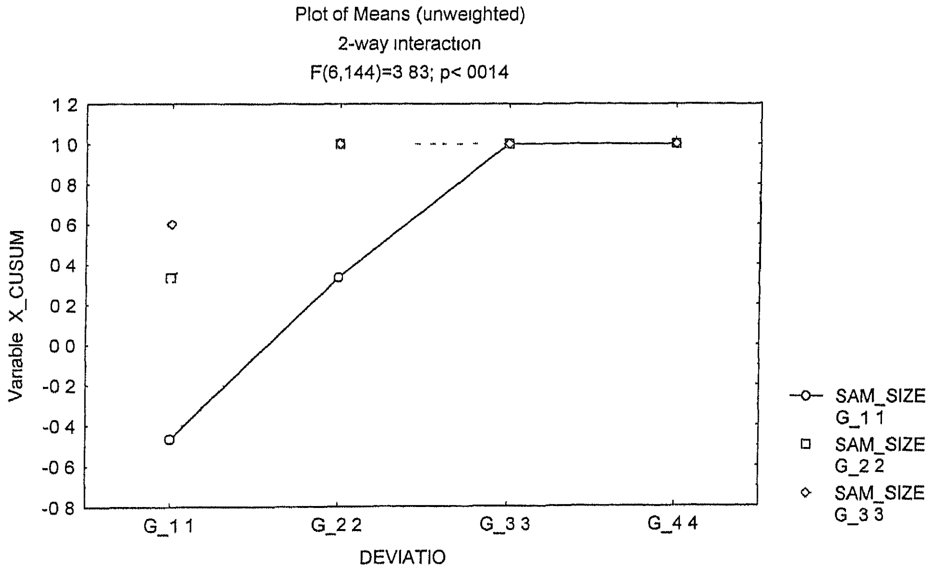

Figure 3.48 X_CUSUM vs. SAMPLE SIZE AND DEVIATION

3.11 Design of Experiment with Run-Length as Performance Measure

This design of experiment is done for two variable case, but run-length (number of samples after which there is signal) are considered as performance measure Levels of factors are same as in the DOE discussed in section-3.9 In case a signal in not observed in 10 samples, run length is taken as 11

Table 3.26 Factors of Design of Experiment

Factors	Levels
Correlation Coefficient	Low, Medium and High
Sample Size	1, 6 and 10
Deviation	4 Combinations of Low, Medium and High

Levels of deviation:

- **Low** $< \pm 0.5$
- **Medium** $> \pm 0.5$ **and** $< \pm 1.0$
- **High** $> \pm 1.0$
- **High and Low Combinations of Deviations**

3.12 Analysis of DOE

From the design of experiment, following results are obtained for different charts.

3.12.1 T^2 CONTROL CHART :

From the design of experiment it can be observed that following factors are significant

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**
- **Correlation Coefficient and Sample Size**

Table 3.27 **Summary of Effects for T^2 Control Chart**

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	95 62222	144	4 061111	23 54583	1 42E-09
2	2	206 0722	144	4 061111	50 74282	2 09E-17
3	3	118 1093	144	4 061111	29 08299	9 22E-15
12	4	22 38889	144	4 061111	5.512996	0 00037
13	6	5 97037	144	4 061111	1 470132	0 192455
23	6	2 42037	144	4 061111	0 595987	0.733166
123	12	3 081481	144	4 061111	0 758778	0 691647

Following are the observations made from the results as shown in Fig-3 49 to Fig-3 54

- **Graphs for this charts are similar to Figure from 3.22 to 3.24. which are conducted by using +1 and –1 as performance measure.**
- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement in the performance is slow.**
- **At high deviation, signals are faster than low deviation.**
- **In two-way interaction effect between correlation coefficient and sample size, at particular sample size, as correlation coefficient increases, performance improves. At n = 1, number of signals are more due to increase in correlation coefficient as compared to n > 1.**
- **When DOE is conducted by using +1 and –1 as performance measure, two-way interaction effect between correlation coefficient and deviation (section-3.10.1) was also significant, which is not here.**

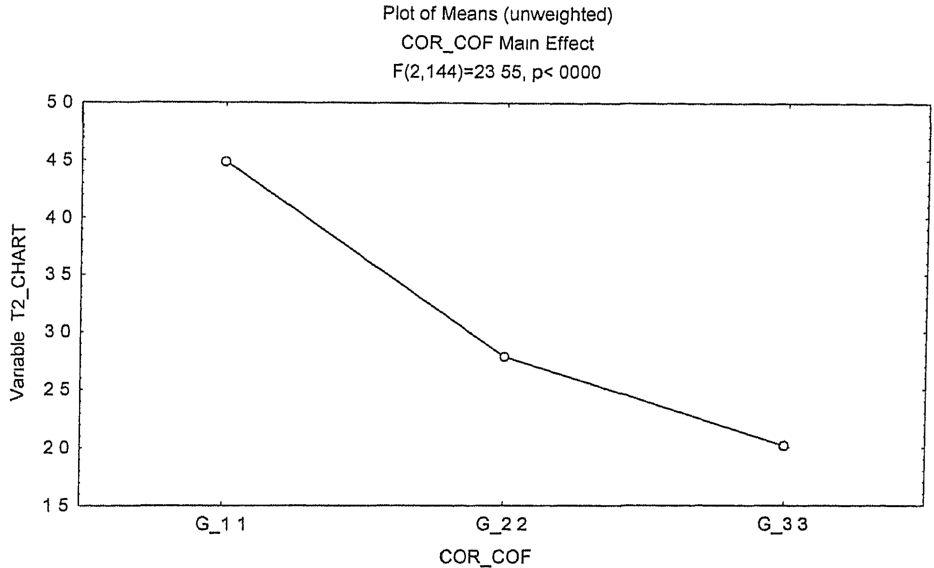

Figure 3.49 T^2 **CHART vs. CORRELATION COEFFICIENT**

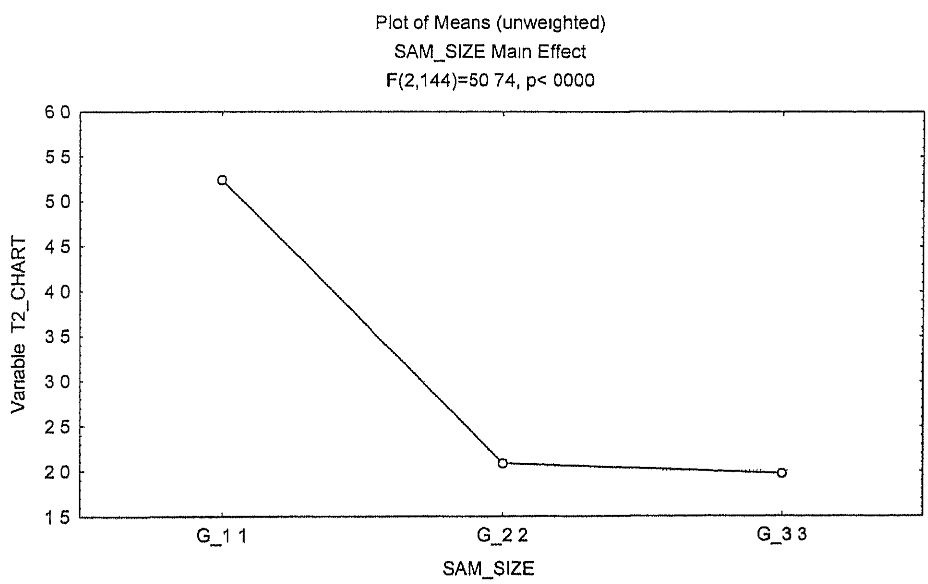

Figure 3.50 T^2 **CHART vs. SAMPLE SIZE**

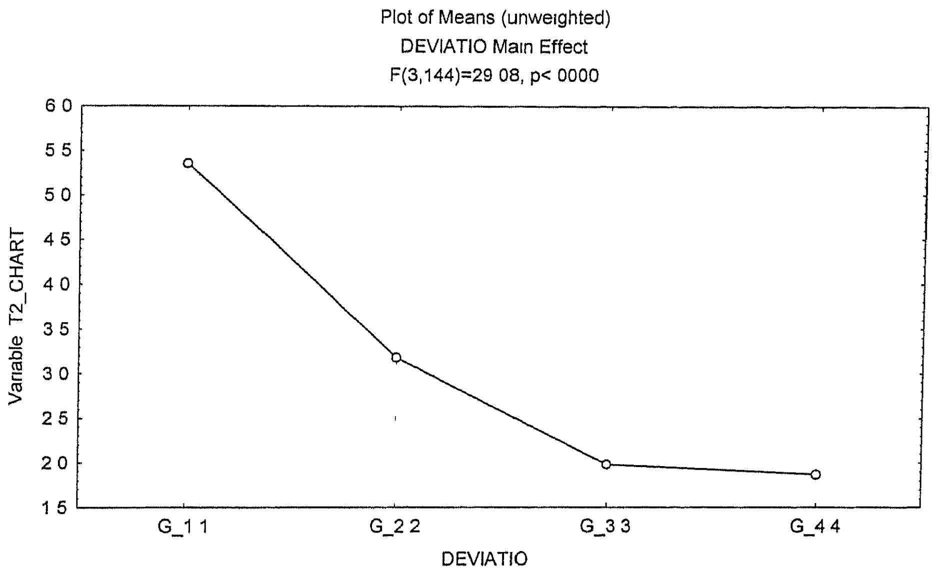

Figure 3.51 T^2 **CHART vs. DEVIATION**

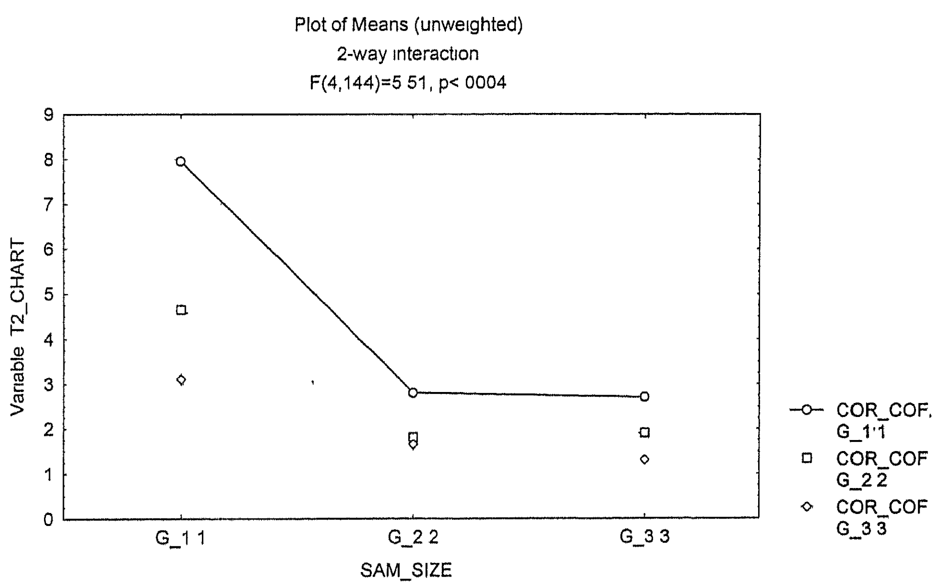

Figure 3.52 T^2 **CHART vs. CORRELATION COEFFICIENT AND SAMPLE SIZE**

3.12.2 MULTIVARIATE CUSUM CONTROL CHARTS

3.12.2.1 CUSUM #1 :

From the design of experiment it can be observed that following factors are significant.

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**

Table 3.28 Summary of Effects for Cusum #1 Chart

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	23 43889	144	3 933333	5 95904	0 003263
2	2	119 2389	144	3 933333	30 31497	1 03E-11
3	3	135 1019	144	3 933333	34 34793	8.32E-17
12	4	1 013889	144	3.933333	0 257768	0 904519
13	6	1 97963	144	3.933333	0 503296	0 805098
23	6	2 846296	144	3 933333	0.723635	0 631221
123	12	0 465741	144	3 933333	0 118409	0 999886

Following are the observations made from the results as shown in Fig-3 53 to Fig-3 55

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance of this chart improves. But after sample size n = 6, improvement in performance is slow.**
- **At high deviation, signals are faster than low deviation.**
- **There are no significant interaction effects on this chart.**

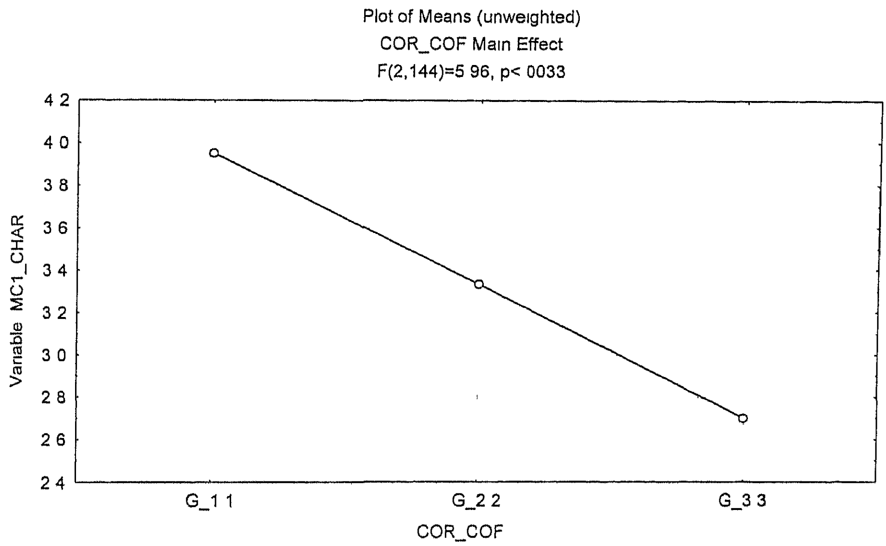

Figure 3.53 CUSUM #1 vs. CORRELATION COEFFICIENT

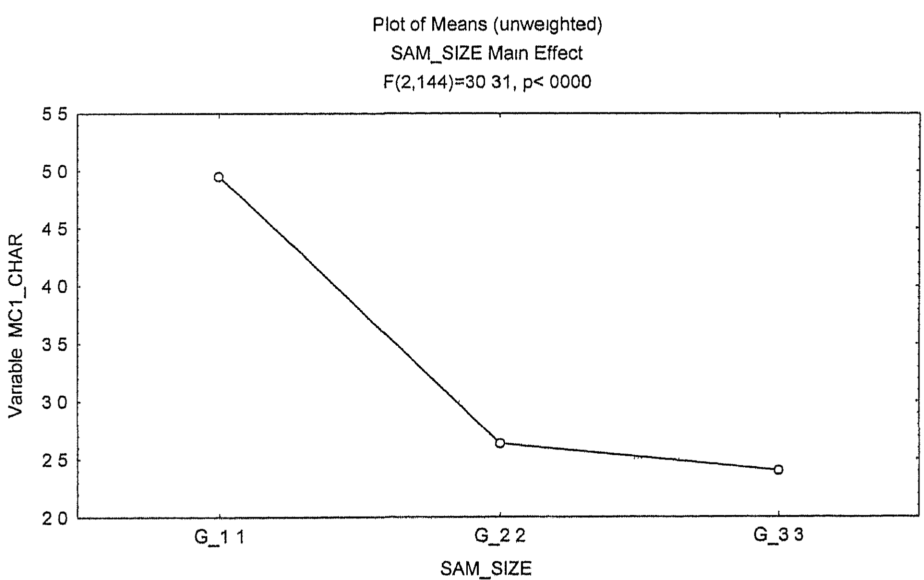

Figure 3.54 CUSUM #1 vs. SAMPLE SIZE

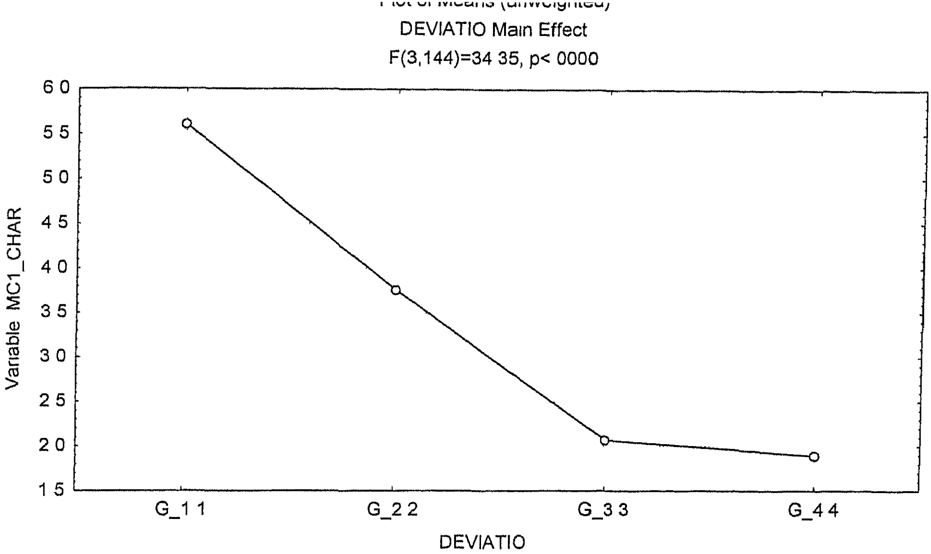

Figure 3.55 CUSUM #1 vs. DEVIATION

3.12.2.2 CUSUM #2:

From the design of experiment it can be observed that following factors are significant

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**

Table 3.29 Summary of Effects for Cusum #2 Chart

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	31 65	144	3 919445	8 075124	0 000474
2	2	231 8	144	3 919445	59 14103	1 78E-19
3	3	113 2889	144	3.919445	28 90432	1 09E-14
12	4	7 925	144	3 919445	2 02197	0 094435
13	6	2 205555	144	3 919445	0 562721	0 759412
23	6	6 622222	144	3 919445	1 689582	0 127578
123	12	1.347222	144	3 919445	0 343728	0.979414

Following are the observations made from the results as shown in Fig-3 56 to Fig-3 58

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement in performance is slow.**
- **At high deviation, signals are faster than low deviation.**

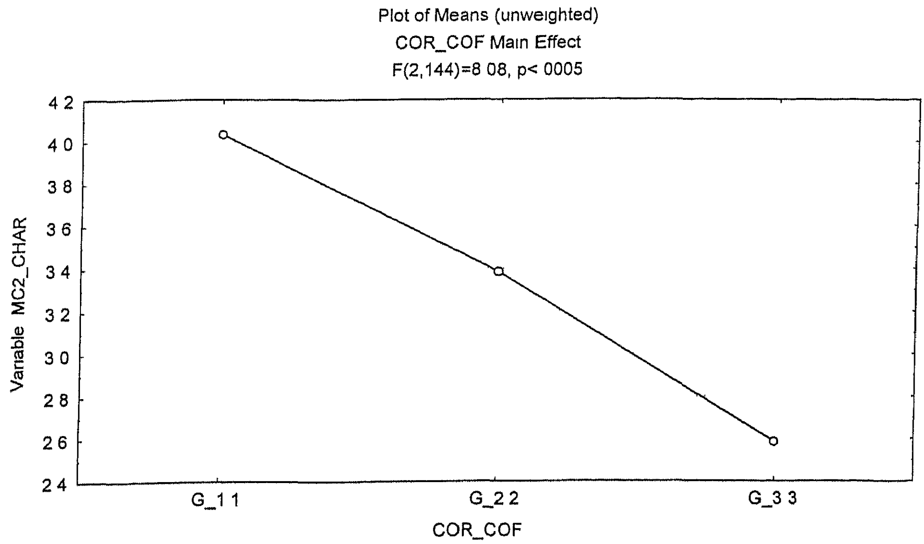

Figure 3.56 CUSUM #2 vs. CORRELATION COEFFIECIENT

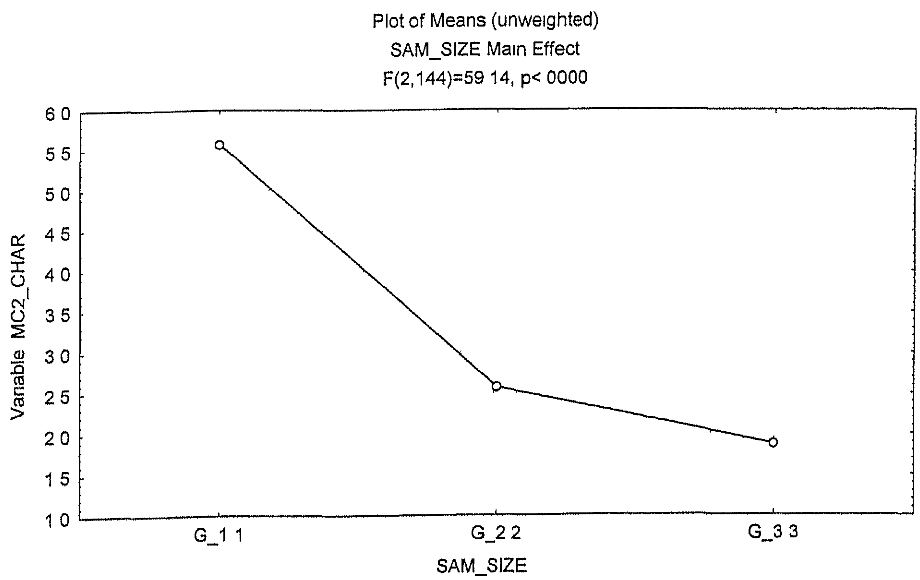

Figure 3.57 Cusum #2 vs. Sample Size

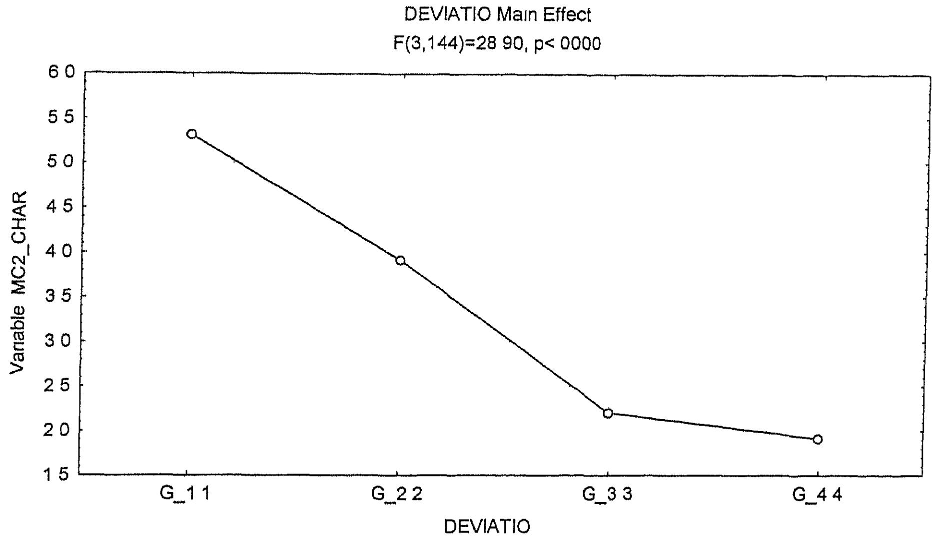

Figure 3.58 **CUSUM #2 vs. DEVIATION**

3.12.3 Z_METHOD :

From the design of experiment it can be observed that following factors are significant

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**

Table 3.30 **Summary of Z_Method Chart**

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	24 06667	144	3 755556	6 408284	0 002157
2	2	134 3167	144	3 755556	35 76479	2 45E-13
3	3	147.087	144	3 755556	39.16519	1 44E-18
12	4	0 933333	144	3 755556	0.248521	0.910172
13	6	3 503704	144	3 755556	0.932939	0 473365
23	6	6 420371	144	3 755556	1 709566	0 12278
123	12	1 57037	144	3 755556	0.418146	0 954494

Following are the observations made from the results as shown in Fig-3 59 to Fig-3 61

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement in performance is slow.**

- **At high deviation, signals are faster than low deviation.**

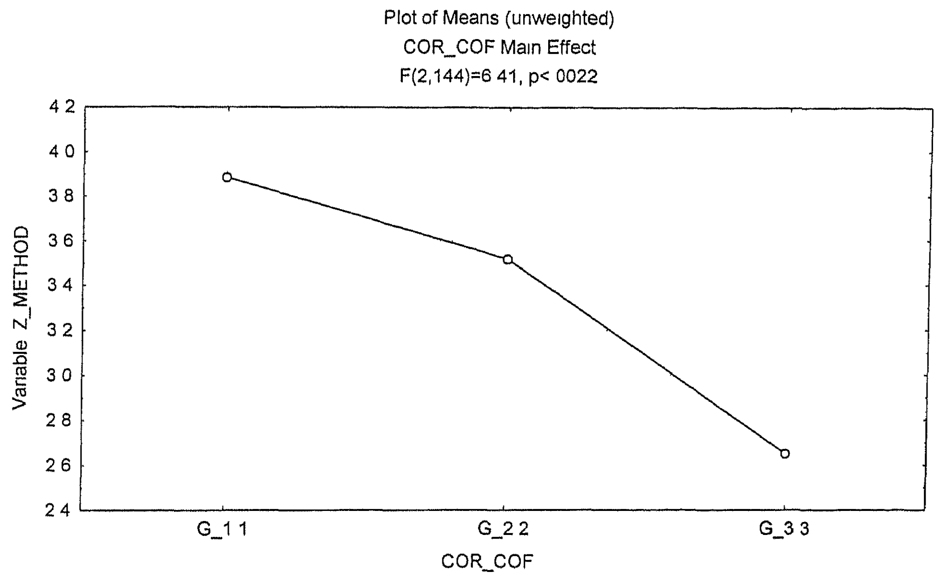

Figure 3.59 Z_METHOD vs. CORRELATION COEFFICIENT

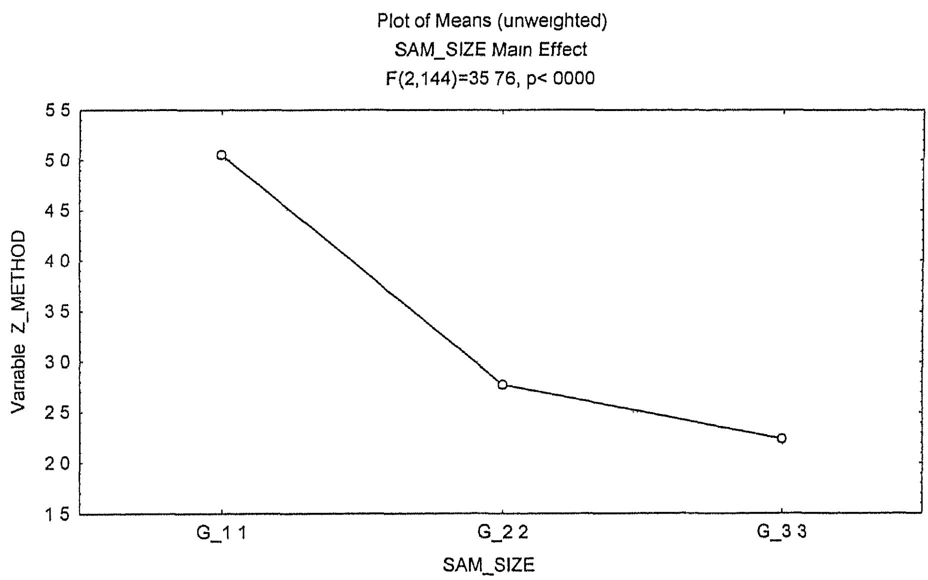

Figure 3.60 Z_METHOD vs. SAMPLE SIZE

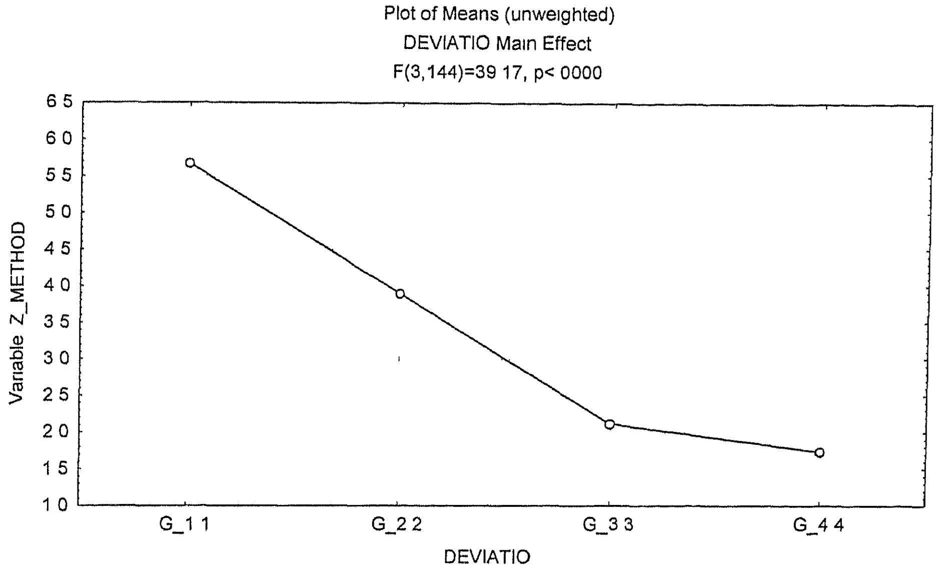

Figure 3.61 **Z_METHOD vs. DEVIATION**

3.12.4 SHEWHART X_BAR CHART :

From the design of experiment it can be observed that following factors are significant.

- **Sample Size**
- **Deviation**
- **Correlation Coefficient and Sample Size**

Table 3.31 **Summary of Effect of X_Bar Chart**

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	10 07222	144	3 755556	2.681953	0 071848
2	2	112 1556	144	3 755556	29 8639	1 41E-11
3	3	252 0352	144	3 755556	67 10996	3 29E-27
12	4	24 80556	144	3.755556	6 60503	6.57E-05
13	6	2 67963	144	3 755556	0.713511	0 639267
23	6	6.540741	144	3 755556	1 741617	0 115428
123	12	5 635185	144	3 755556	1 500493	0 13023

Following are the observations made from the results as shown in Fig-3 62 to Fig-3 65

- **With the increase in correlation coefficient, this chart performs better.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement in performance is not much.**
- **At high deviation, signals are faster than low deviation.**

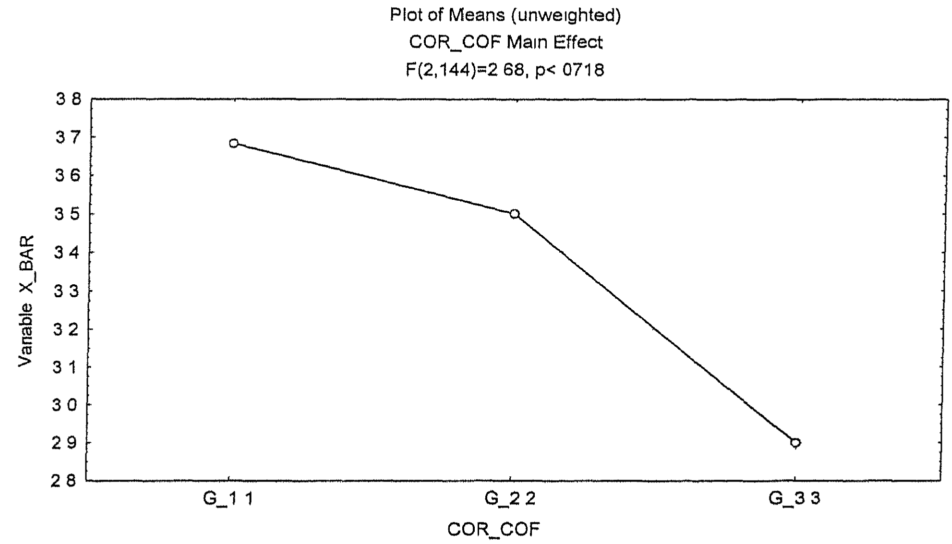

Figure 3.62 X_BAR CHART vs. CORRELATION COEFFICIENT

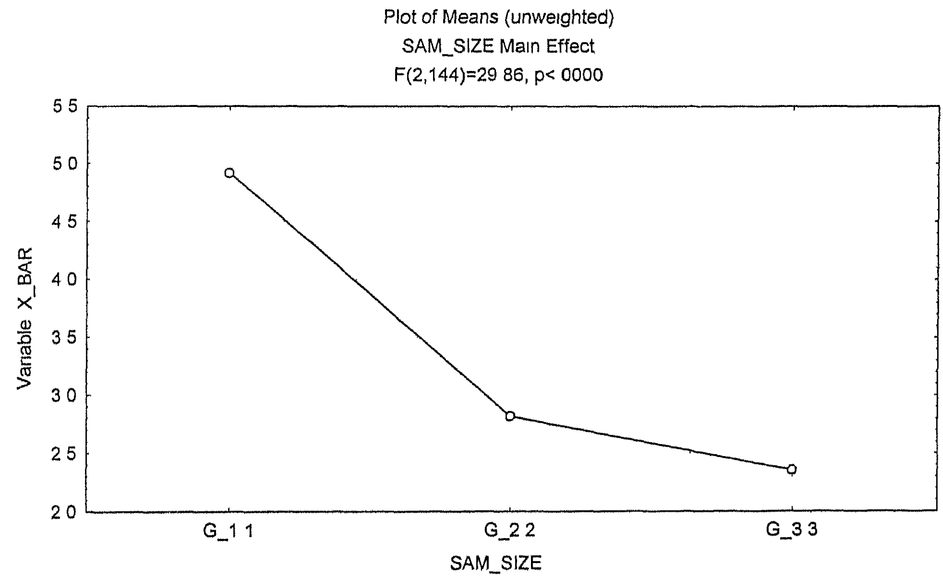

Figure 3.63 X_BAR CHART vs. SAMPLE SIZE

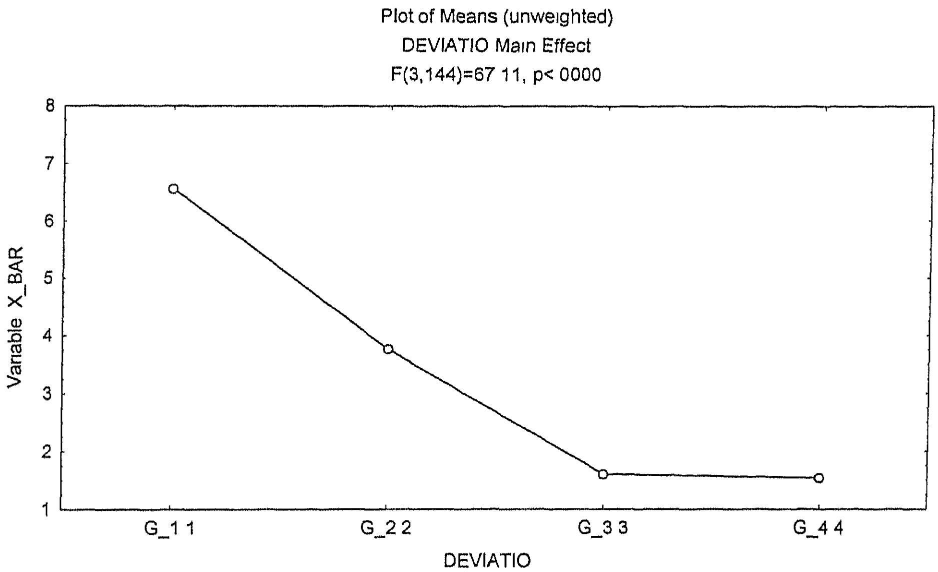

Figure 3.64 X_BAR CHART vs. DEVIATION

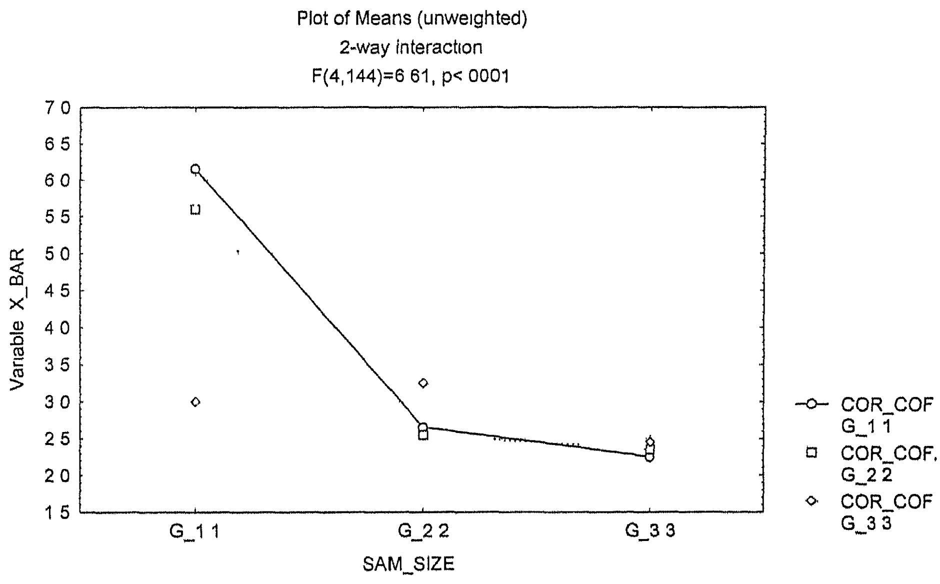

Figure 3.65 X_BAR CHART vs. CORRELATION COEFFICIENT AND SAMPLE SIZE

3.12.4 UNIVARIATE X_CUSUM CHART :

From the design of experiment it can be observed that following factors are significant

- **Sample Size**
- **Deviation**

Table 3.32 **Summary of Effects of X_Cusum Chart**

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	54 42222	144	49.90556	1 090504	0 338806
2	2	319 0722	144	49.90556	6.393521	0 002187
3	3	471 6593	144	49 90556	9 451037	9 68E-06
12	4	58 44722	144	49 90556	1.171157	0 326057
13	6	54 1037	144	49 90556	1 084122	0 374739
23	6	51 97593	144	49.90556	1.041486	0 401012
123	12	46 66204	144	49 90556	0 935007	0.514044

Following are the observations made from the results as shown in Figure 3.66 to 3 70

- **With the increase in correlation coefficient, this chart performance declines but after second level, performance again starts to improve.**
- **As sample size n increases, performance increases. But after sample size n = 6, improvement in performance is slow.**
- **At high deviation, signals are faster than low deviation.**

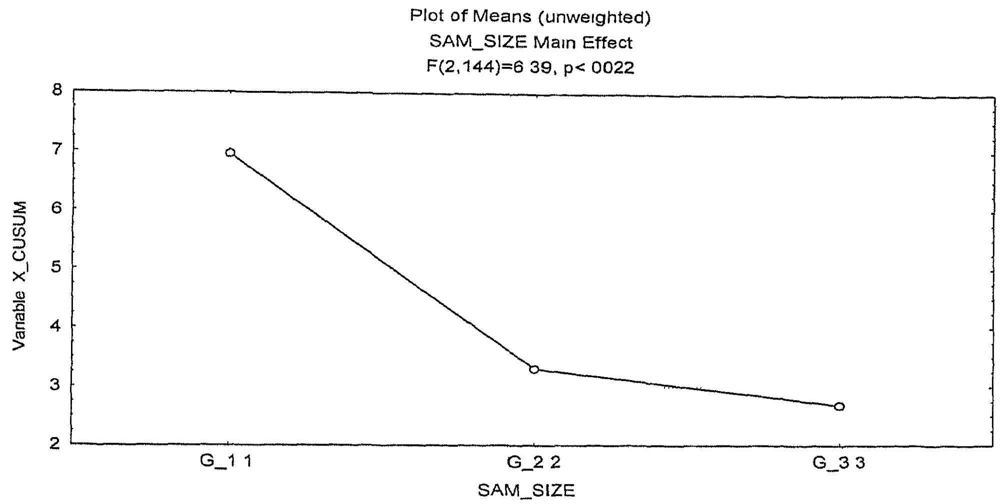

Figure 3.66 **X_CUSUM vs. SAMPLE SIZE**

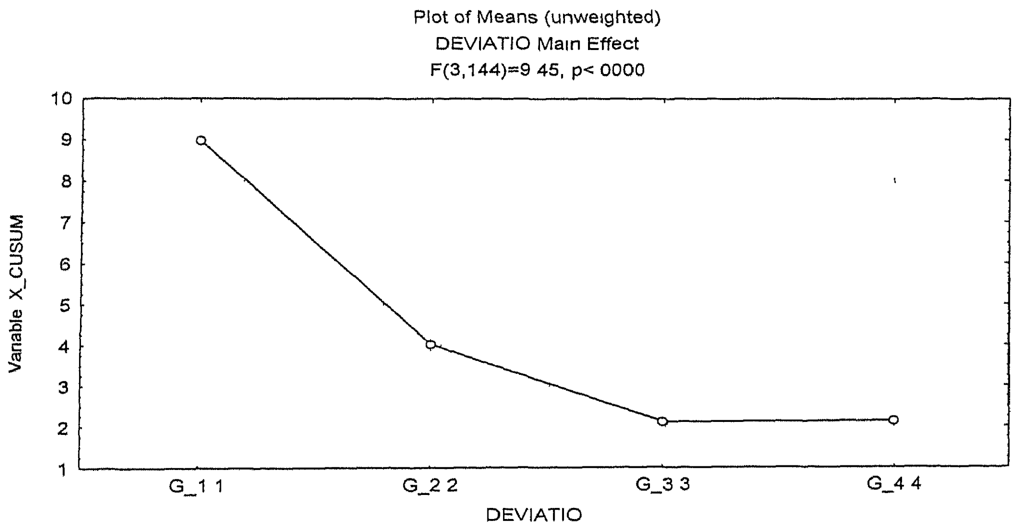

Figure 3.67 **X_CUSUM vs. DAVIATION**

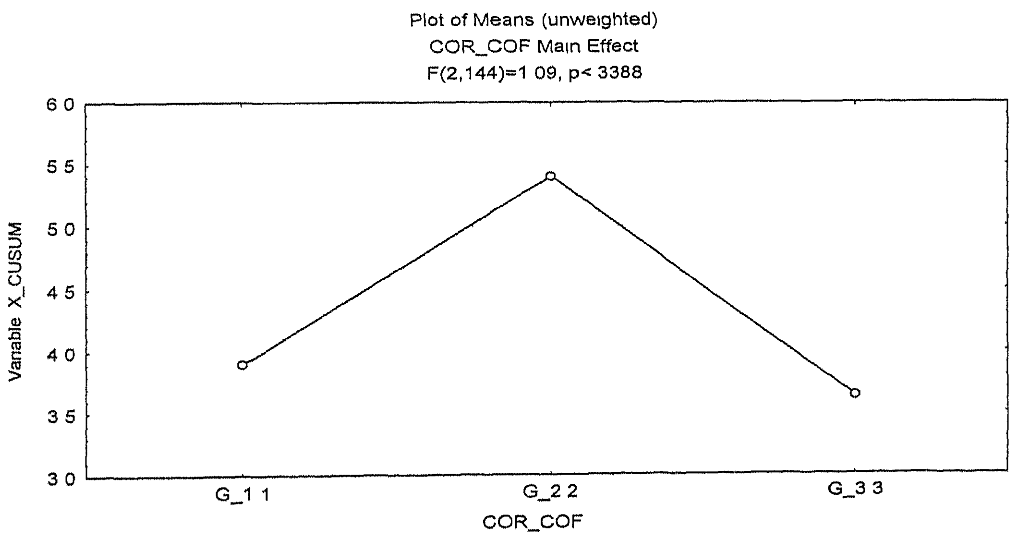

Figure 3.68 **X_CUSUM vs. CORRLATION COEFFICIENT**

3.13 Design of Experiment for Five Variables with Run-Length as Performance Measure

Performance of charts is also examined on five variables to see the impact of deviation w.r.t to number of variable. This design of experiment is also conducted for 3 levels of correlation coefficient, 3 levels of sample size and 4 levels for deviations. The run length is taken as output for each combination of these factors.

Following are the factors and levels for this design of experiment.

Table 3.33 **Factors of DOE**

Factor	Levels
Correlation Coefficient	**Low, Middle and High**
Sample Size	**1,6 and 10**
Deviation	**Low, Middle, High and Combination of Low and High**

In case of five variables, the correlation coefficient between variables are discussed in the section-3.1. The levels of deviation are alike two variable.

3.14 Analysis of DOE

Six charts are analyzed for 5 variable case. This experiment is replicated 7 times for all charts. Thus total of 256 (3 * 3 * 4 * 7) problems are solved for these charts as described below.

3.14.1 T^2 CONTROL CHART :

From the design of experiment it can be observed that following factors are significant:

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**

- **Correlation Coefficient and Sample Size**
- **Correlation Coefficient and Deviation**
- **Sample Size and Deviation**
- **Correlation Coefficient, Sample Size and Deviation**

Table 3.34 **Summary of Effects of T^2 Control Chart**

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	134.9184	216	6 775215	19.91352	1.16E-08
2	2	263.0822	216	6.775215	38.83009	3.92E-15
3	3	391.1208	216	6.775215	57.72817	1.91E-27
12	4	29.8489	216	6.775215	4.405602	0.001917
13	6	21.89553	216	6.775215	3.23171	0.004628
23	6	18.07892	216	6.775215	2.668391	0.016154
123	12	12.8137	216	6.775215	1.891261	0.036709

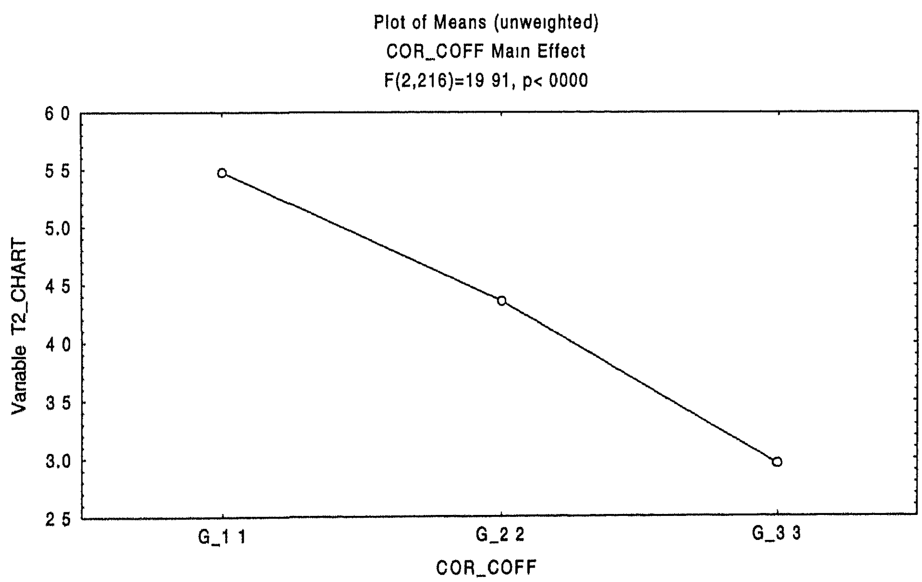

Figure 3.69 T^2 **CHART vs. CORRELATION COEFFICIENT**

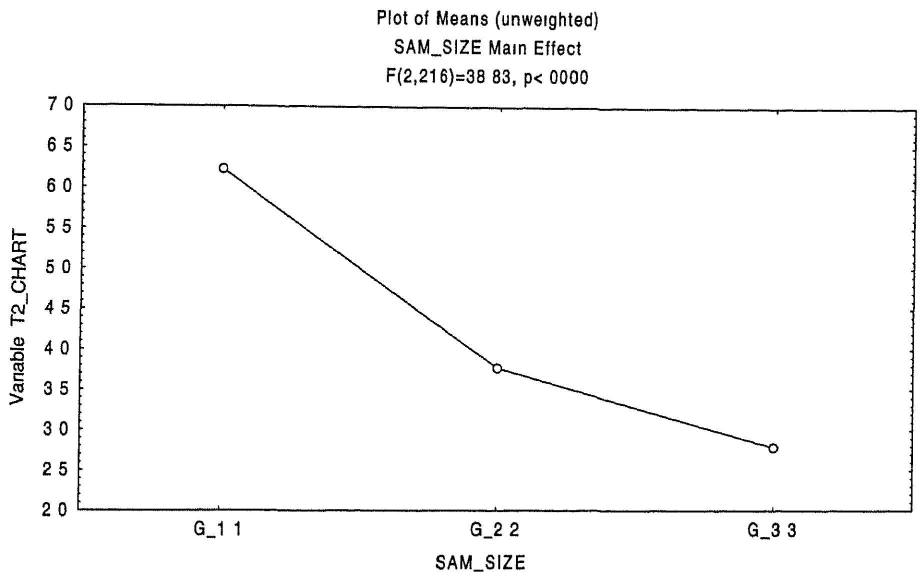

Figure 3.70 T^2 **CHART vs. SAMPLE SIZE**

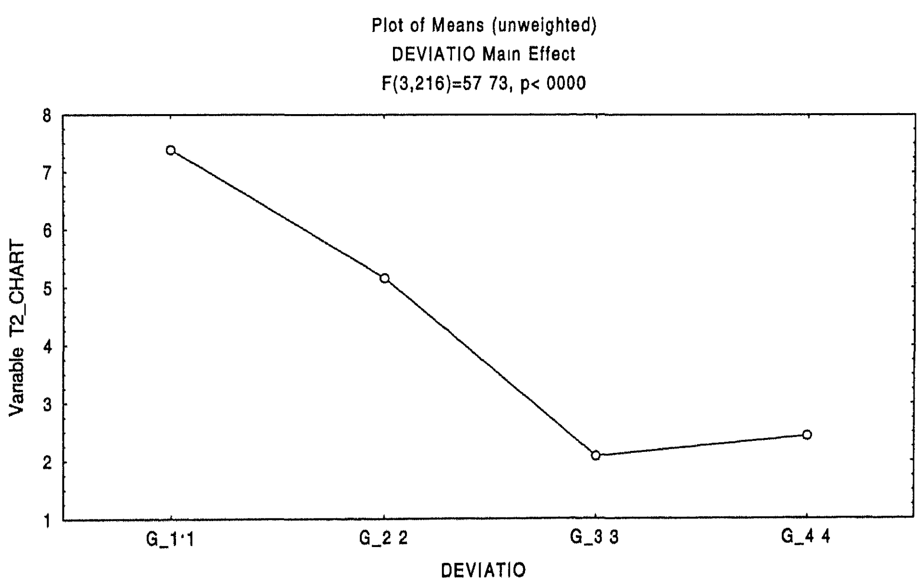

Figure 3.71 T^2 **CHART vs. DEVIATION**

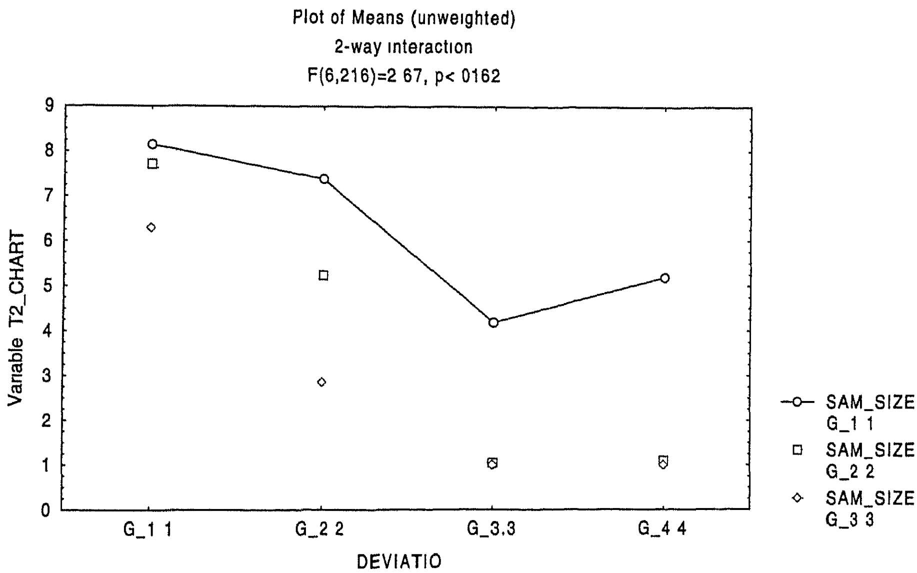

Figure 3.72 T^2 **CHART vs. DEVIATION AND SAMPLE SIZE**

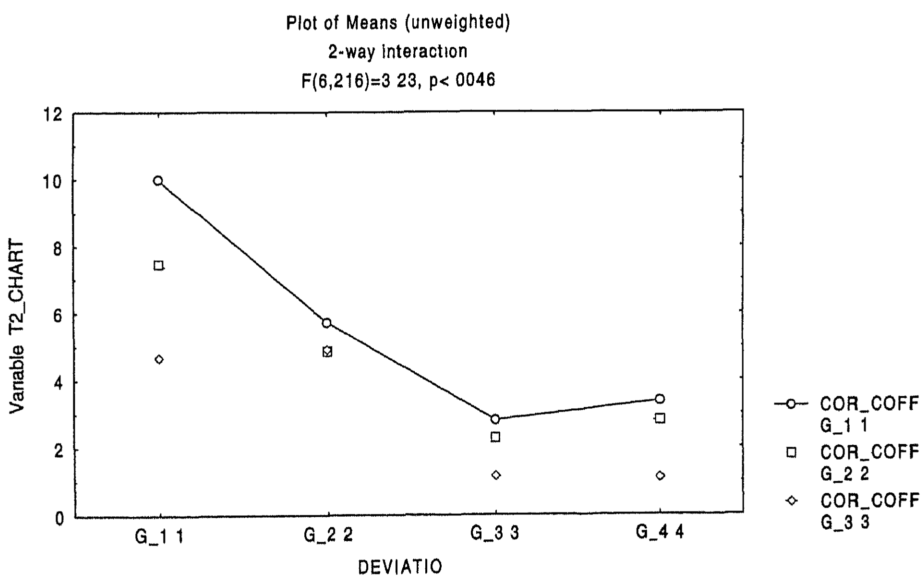

Figure 3.73 T^2 **CHART vs. DEVIATION AND CORRELATION COEFFICIENT**

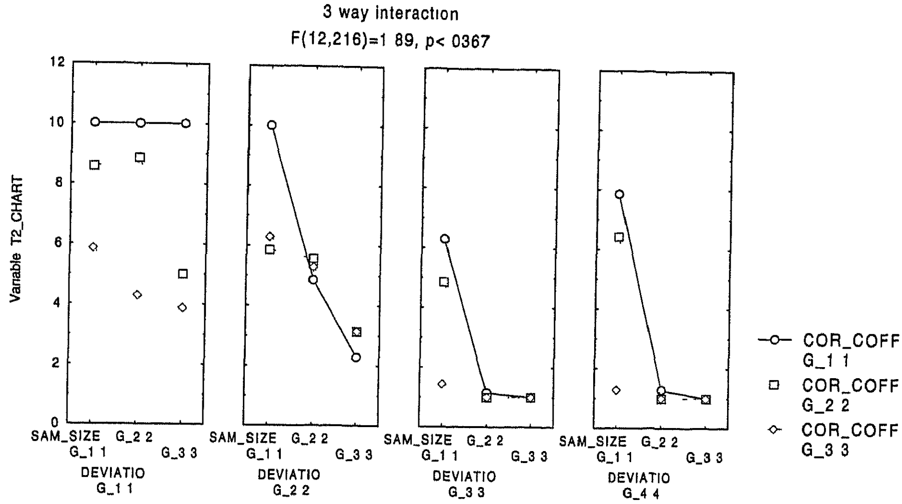

Figure 3.74 T^2 **CHART vs. DEVIATION, CORRELATION COEFFICIENT AND SAMPLE SIZE**

3.14.2 MULTIVARIATE CUSUM CHARTS

3.14.2.1 CUSUM #1:

From the design of experiment it can be observed that following factors are significant:

- **Correlation Coefficient**
- **Sample Size**
- **Deviation**
- **Correlation Coefficient and Sample Size**

Table 3.35 Summary of Effects for Cusum #1

Effects	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	95.91521	216	4.576279	20.95921	4.8E-09
2	2	59.68033	216	4.576279	13.04124	4.5E-06
3	3	257.7311	216	4.576279	56.31892	6.18E-27
12	4	30.72733	216	4.576279	6.71448	4.12E-05
13	6	5.515128	216	4.576279	1.205156	0.304698
23	6	1.866627	216	4.576279	0.407892	0.873355
123	12	2.555474	216	4.576279	0.558417	0.873637

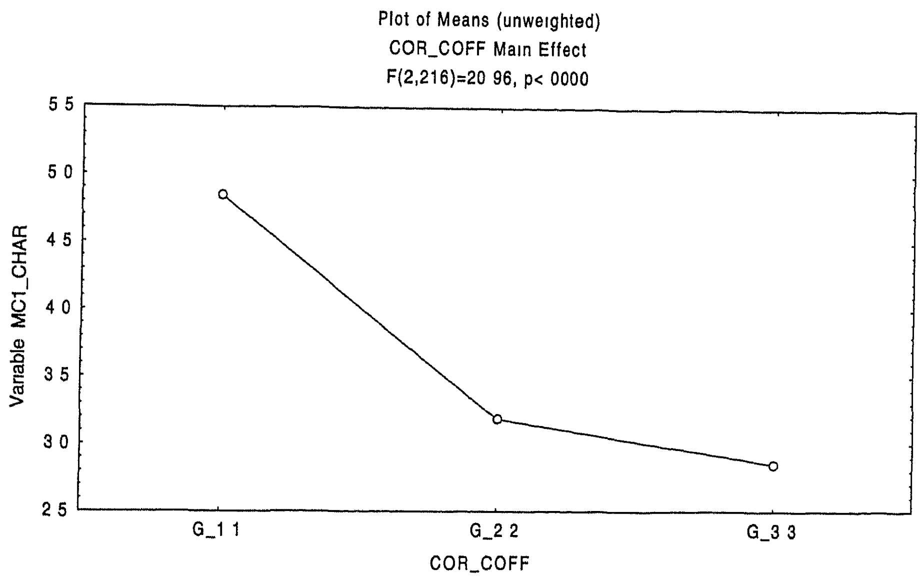

Figure 3.75 CUSUM #1 CHART vs. CORRELATION COEFFICIENT

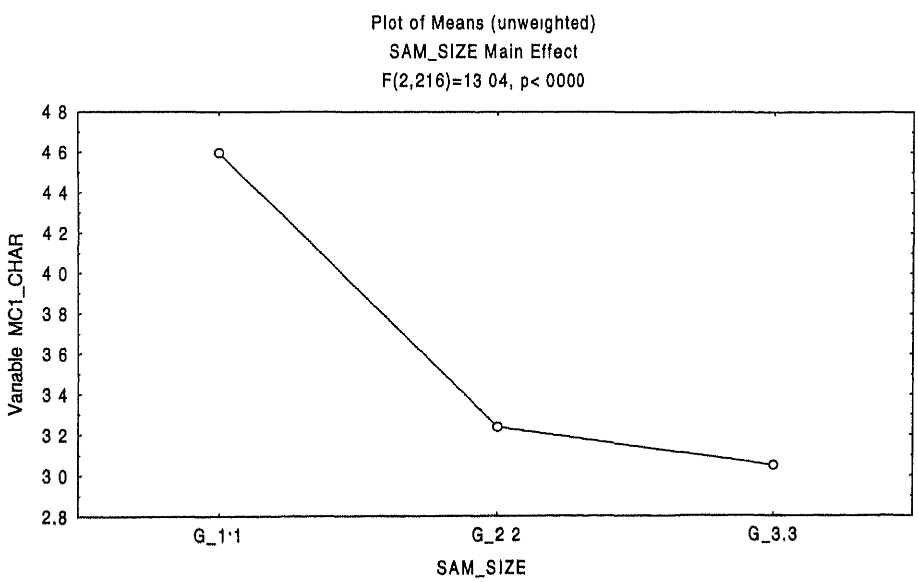

Figure 3.76 CUSUM #1 CHART vs. SAMPLE SIZE

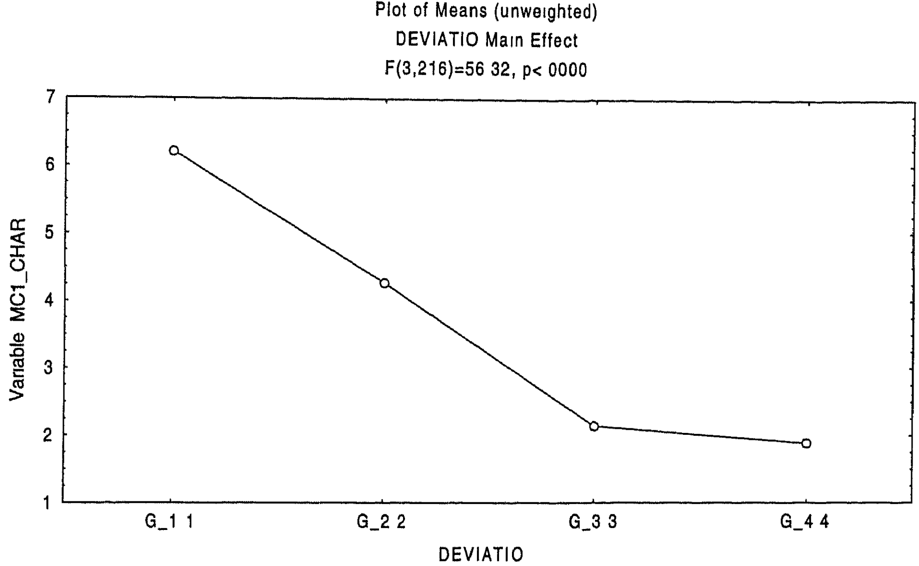

Figure 3.77 **CUSUM #1 CHART vs. DEVIATION**

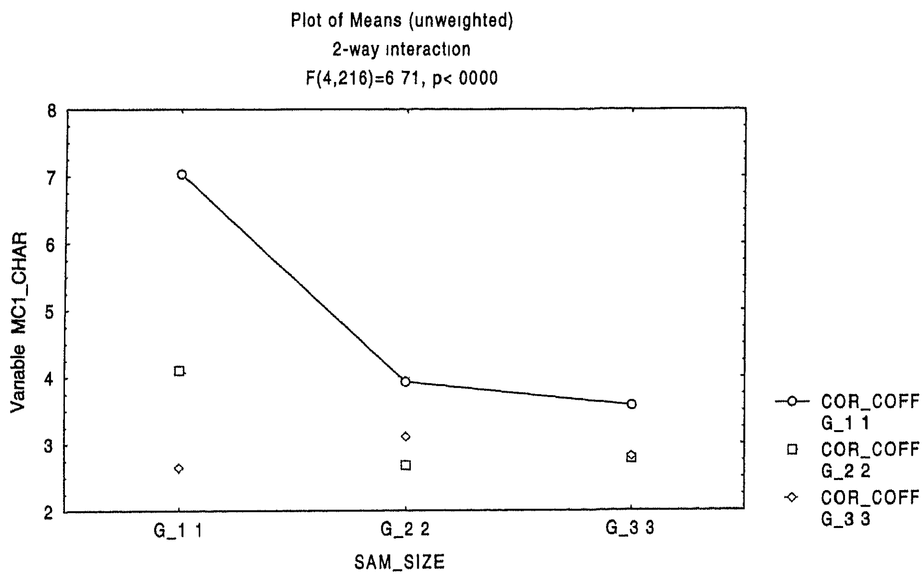

Figure 3.78 **CUSUM #1 CHART vs. SAMPLE SIZE AND CORRELATION COEFFICIENT**

3.14.2.2 CUSUM #2:

From the design of experiment it can be observed that following factors are significant:

- **Correlation Coefficient**
- **Sample Size**

- **Deviation**
- **Correlation Coefficient and Sample size**
- **Correlation Coefficient and Deviation**

Table 3.36 **Summary of Effects for Cusum #2**

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	223.9604	216	2.61993	85.48335	4.49E-28
2	2	29.95492	216	2 61993	11.43348	1.91E-05
3	3	156.8853	216	2.61993	59.88152	3.26E-28
12	4	7.288027	216	2.61993	2.781765	0.027713
13	6	42.94723	216	2.61993	16.39251	1.53E-15
23	6	3.444307	216	2.61993	1.314656	0.251695
123	12	4.69612	216	2.61993	1.79246	0.050775

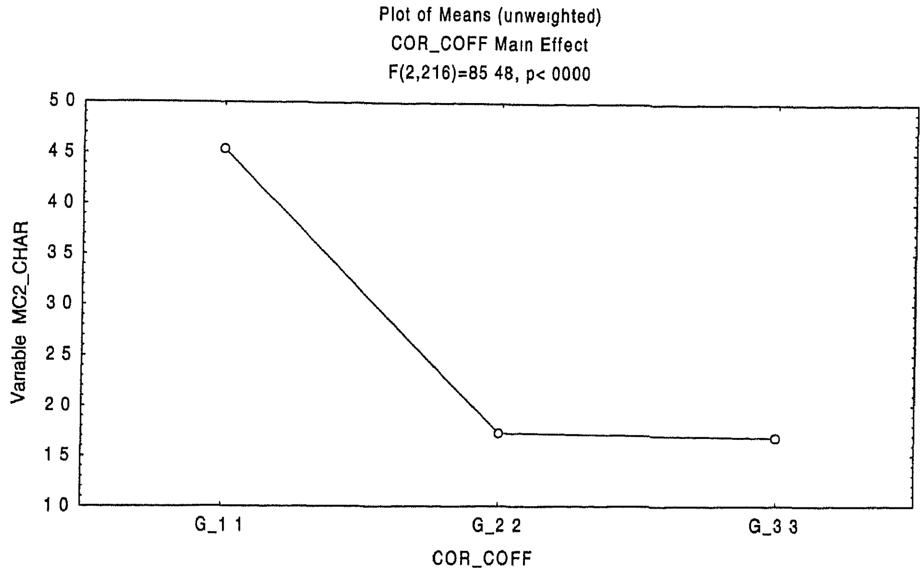

Figure 3.79 CUSUM #2 vs. CORRELATION COEFFICINT

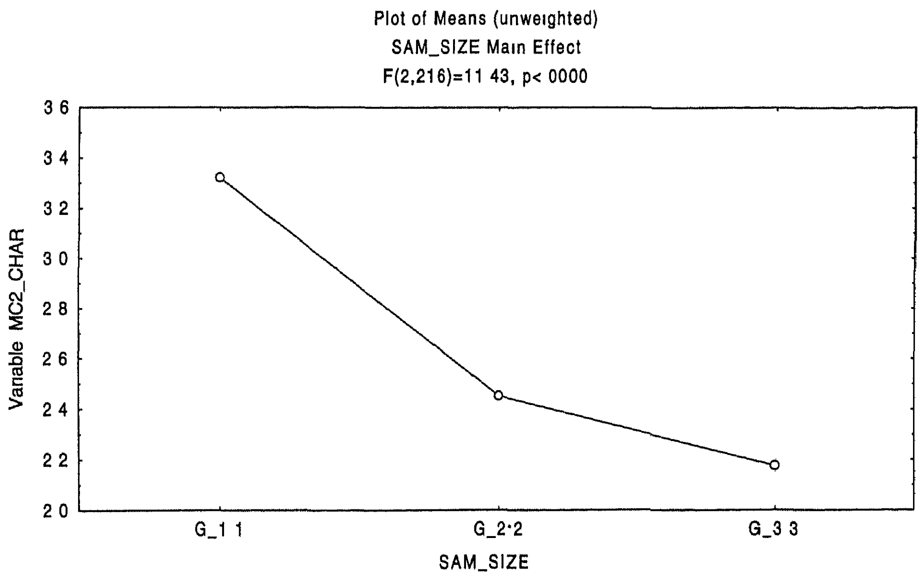

Figure 3.80 CUSUM #2 vs. SAMPLE SIZE

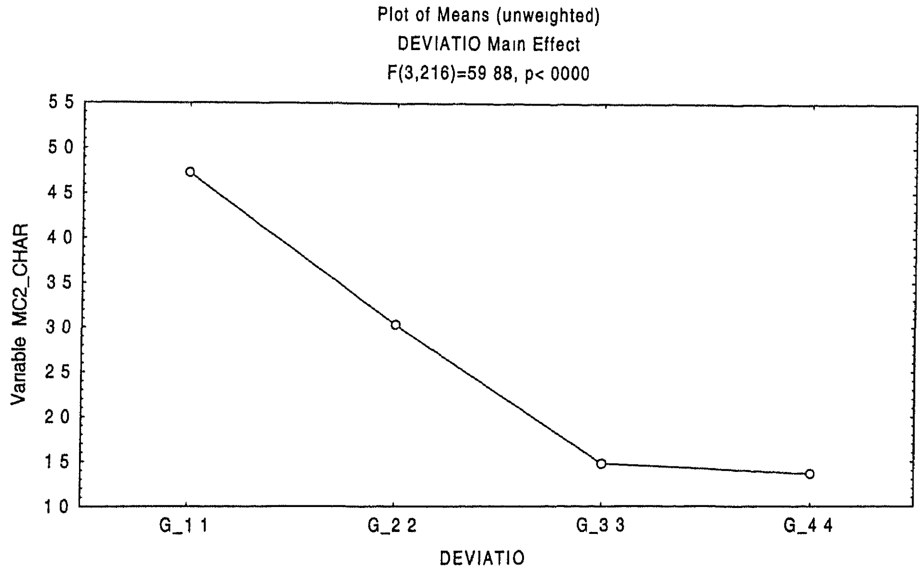

Figure 3.81 **CUSUM #2 vs. DEVIATION**

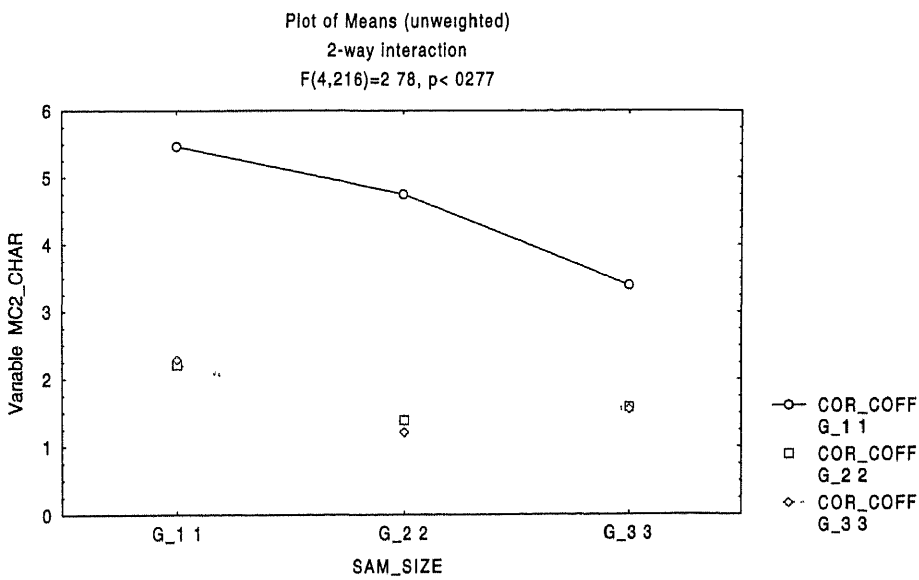

Figure 3.82 **CUSUM #2 vs. CORRELATION COEFFICEINT AND SAMPLE SIZE**

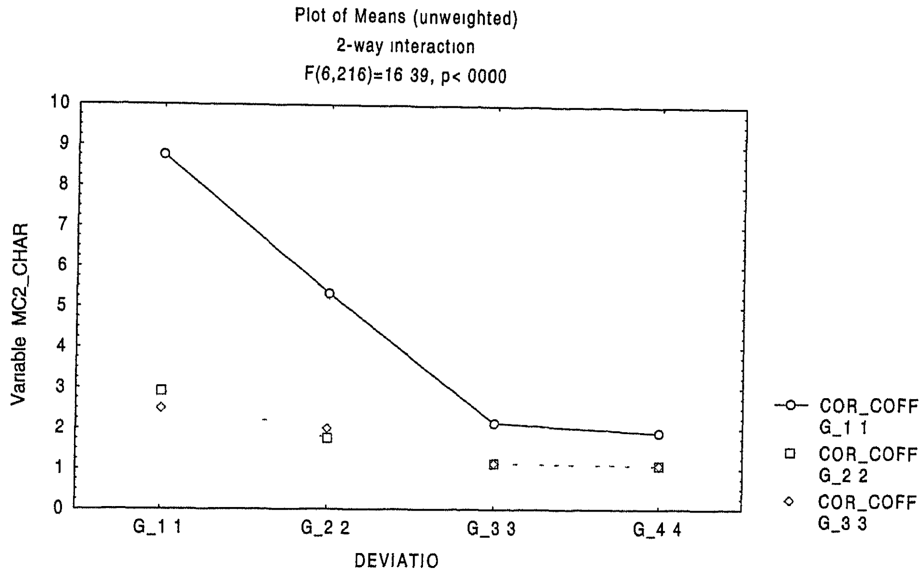

Figure 3.83 CUSUM #2 vs. CORRELATION COEFFICEINT AND DEVIATION

3.14.3 Z_METHOD :

From the design of experiment it can be observed that following factors are significant:

- **Correlation Coefficient**
- **Deviation**
- **Correlation Coefficient and Sample Size**
- **Correlation Coefficient and Deviation**

Table 3.37 Summary of Effects for Z_Method

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	46.96827	216	6.238095	7.529263	0.00069
2	2	7.045756	216	6.238095	1.129472	0.325105
3	3	173.0808	216	6.238095	27.74578	3.23E-15
12	4	35.16052	216	6.238095	5.636419	0.000247
13	6	18.18348	216	6.238095	2.914908	0.009388
23	6	1.665526	216	6.238095	0.266993	0.9518
123	12	3.417421	216	6.238095	0 547831	0.881389

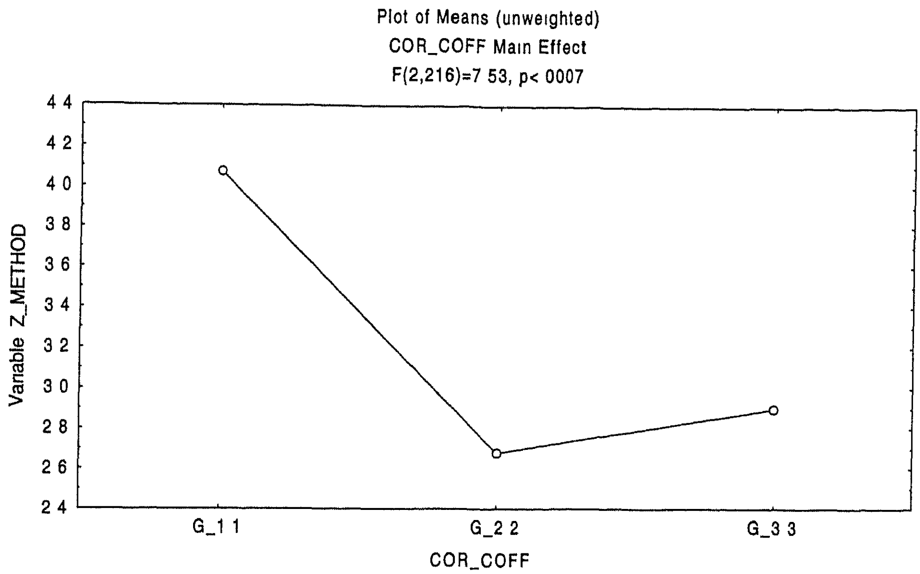

Figure 3.84 **Z_METHOD vs. CORRELATION COEFFICIENT**

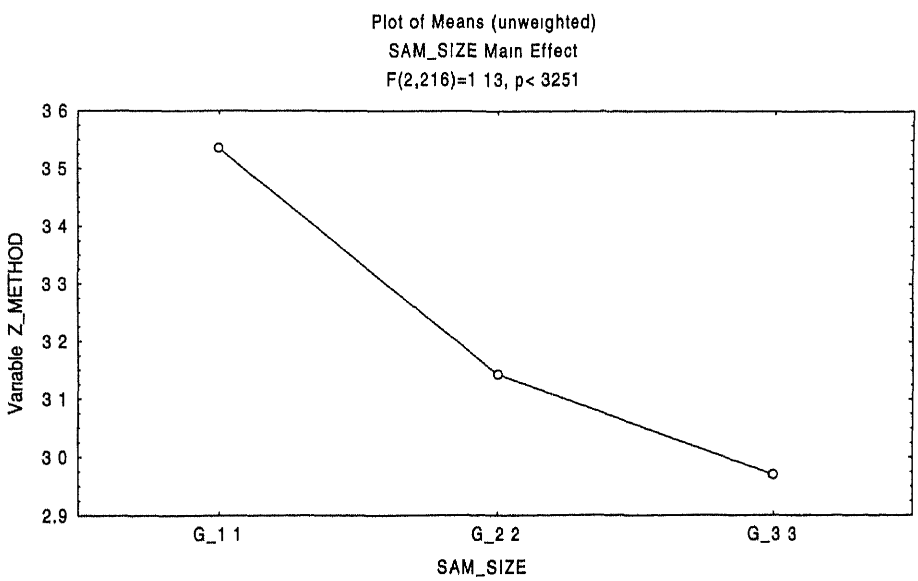

Figure 3.85 **Z_METHOD vs. SAMPLE SIZE**

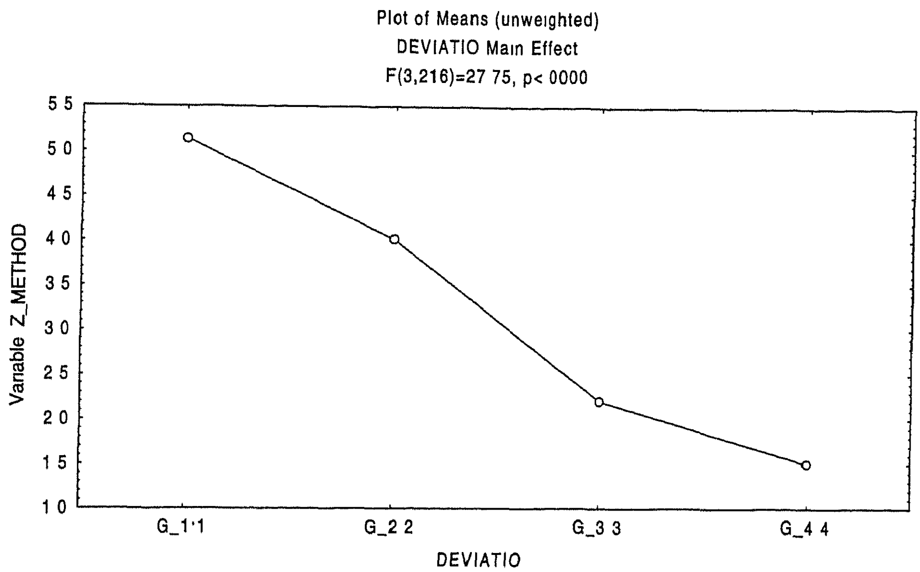

Figure 3.86 **Z_METHOD vs. DEVIATION**

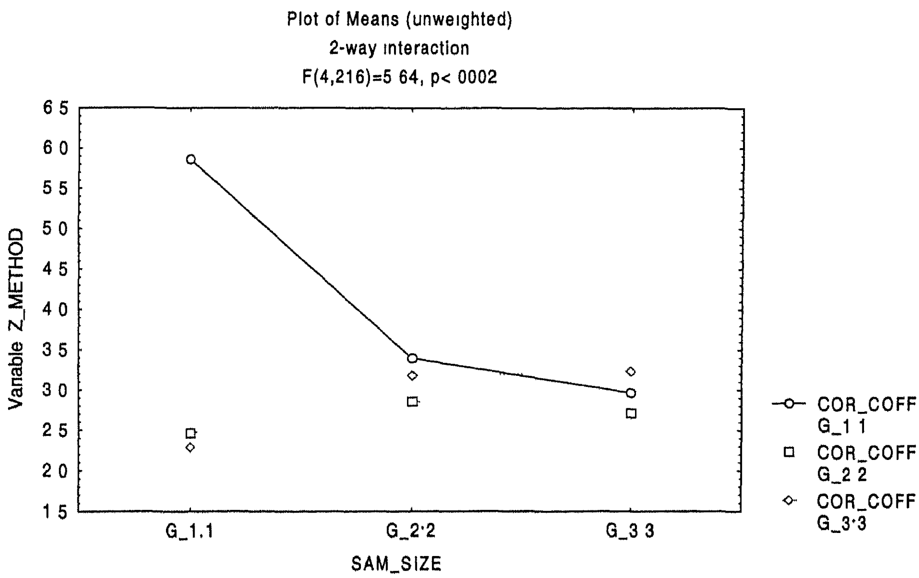

Figure 3.87 **Z_METHOD vs. SAMPLE SIZE AND CORRELATION COEFFICEINT**

3.14.3 SHEWHART X_BAR CHART:

From the design of experiment it can be observed that following factors are significant:

- **Correlation Coefficient**
- **Deviation**
- **Correlation Coefficient and Sample Size**
- **Correlation Coefficient and Deviation**

Table 3.38 Summary of Effects for X_Bar Chart

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	9.133263	216	6.308422	1.447789	0.237362
2	2	347.6745	216	6.308422	55.11276	4.58E-20
3	3	517.1519	216	6.308422	81.97803	0
12	4	6.519671	216	6.308422	1.033487	0.390828
13	6	2.358357	216	6.308422	0.373843	0.895118
23	6	14.8939	216	6.308422	2.360956	0.031398
123	12	3.650069	216	6.308422	0.578603	0.858213

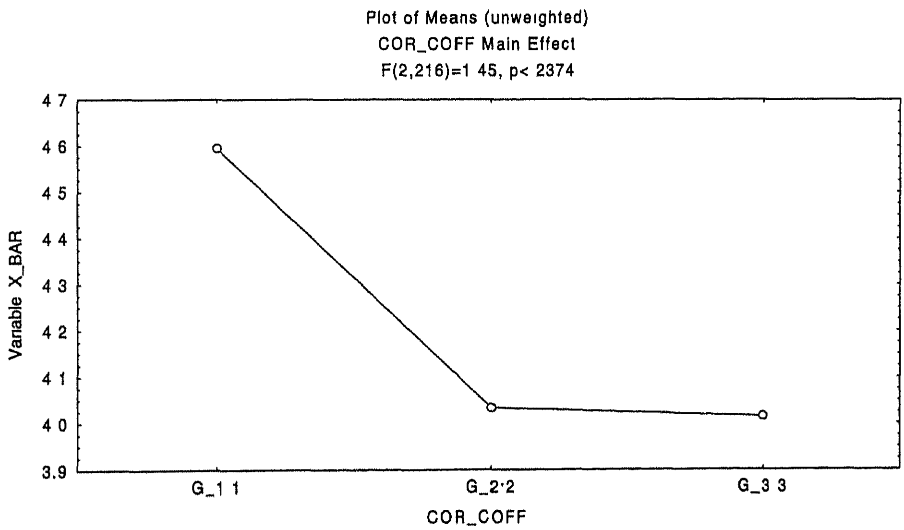

Figure 3.88 X_BAR CHART vs. CORRELATION COEFFICIENT

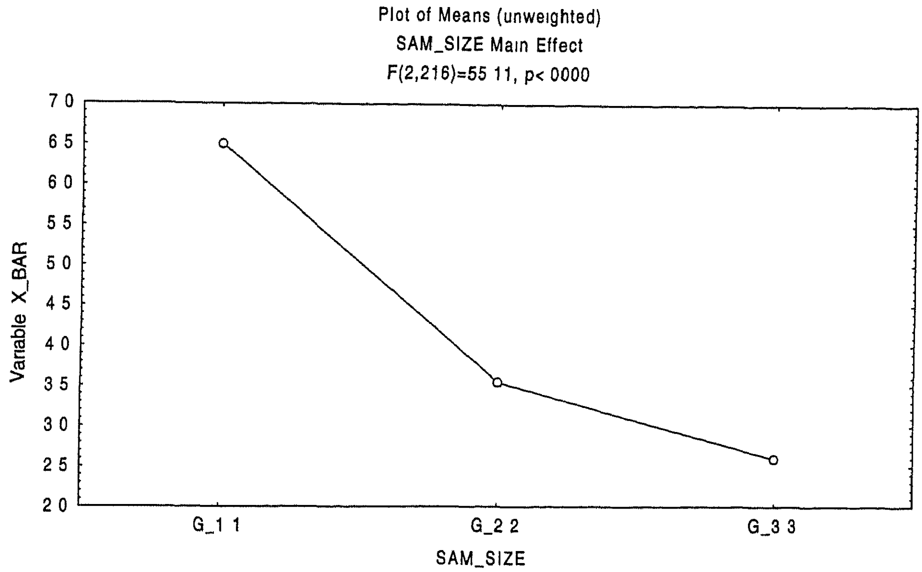

Figure 3.89 **X_BAR CHART vs. SAMPLE SIZE**

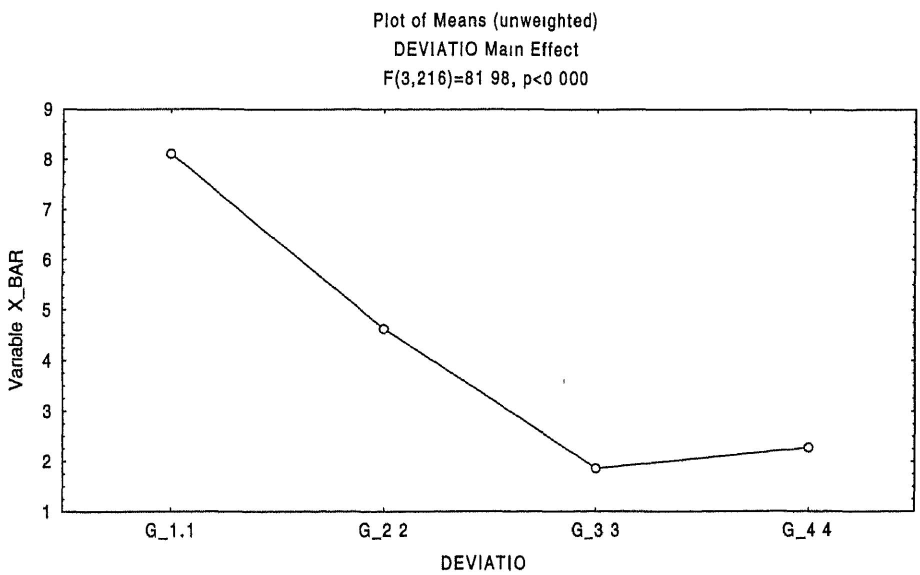

Figure 3.90 **X_BAR CHART vs. DEVIATION**

3.14.4 UNIVARIATE X_CUSUM CHART :

From the design of experiment it can be observed that following factors are significant:

- **Sample Size**
- **Deviation**

Table 3.39 Summary of Effects for X_Cusum Chart

Effect	df Effect	MS Effect	df Error	MS Error	F	p-level
1	2	3.383108	216	5.118937	0.6609	0.517427
2	2	220.1268	216	5.118937	43.00245	1.9E-16
3	3	305.7402	216	5.118937	59.72729	3.7E-28
12	4	2.328756	216	5.118937	0.45493	0.768737
13	6	1.083936	216	5.118937	0.21175	0.972832
23	6	4.596652	216	5.118937	0.89797	0.497215
123	12	1.626521	216	5.118937	0.317746	0.985725

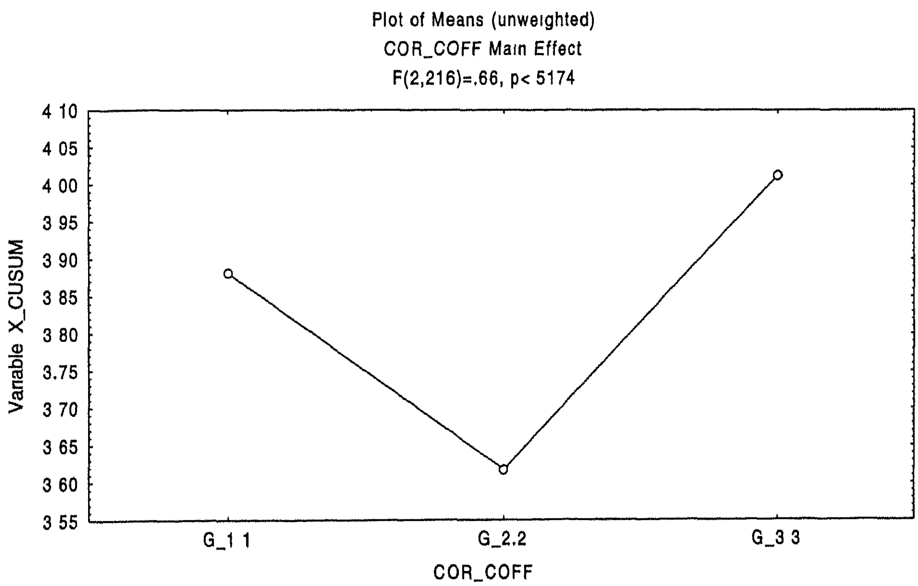

Figure 3.91 X_CUSUM vs. CORRELATION COEFFICIENT

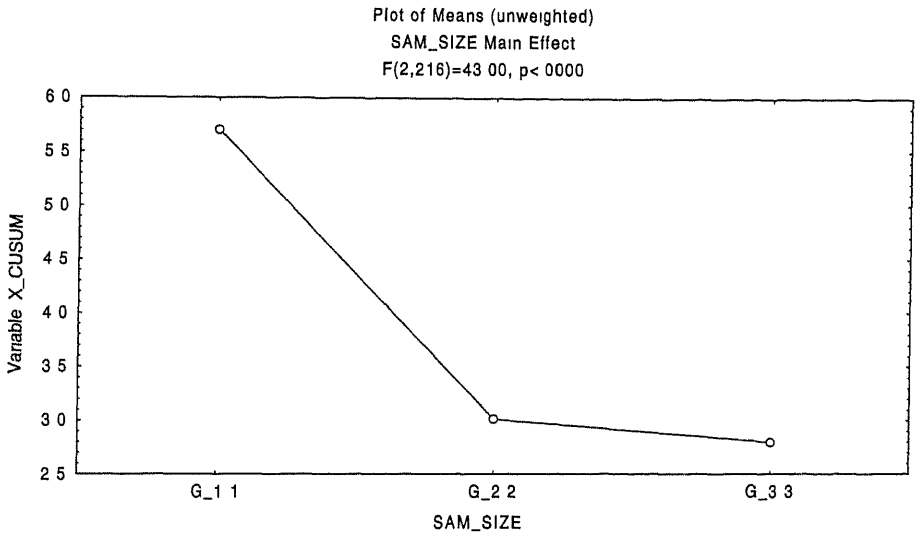

Figure 3.92 **X_CUSUM CHART vs. SAMPLE SIZE**

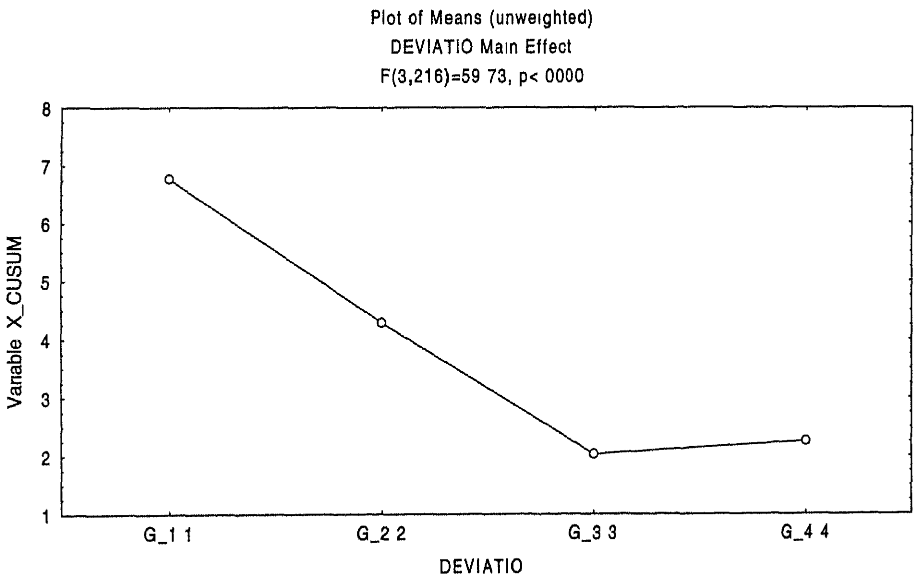

Figure 3.93 **X_CUSUM CHART vs. DEVIATION**

RESULTS AND CONCLUSIONS

This chapter discusses the results obtained after the complete analysis of all charts for a number of problems and the conclusions that can be drawn form the analysis.

4.1 Results and Conclusions

- Charts which is basically for detecting process mean deviation or process variance deviation gives signal when process variance deviation or process mean deviation is introduced
- Performance of following charts increases as correlation coefficient increases
  1. T^2 Control Chart
  2. χ^2 Control Chart
  3. Cusum #1 Chart
  4. Z_Method
  5. X_Bar Chart
  6. Exponentail_Chart

- Performance of Cusum #2 Chart initially increases as correlation coefficient increases but after 0.7 ρ, its becomes constant
- Performance of univariate X_Cusum chart declines as correlation coefficient increases
- Performance of following chart improves as sample size increases
  1. Cusum #1 Chart
  2. X_Bar Chart
  3. Univariate X_Cusum

- Performance of following chart improves as sample size increases but after sample size n = 6, its becomes constant

    1 T^2 Control Chart

    2 χ^2 Control Chart

    3 Cusum #2 Chart

- Performance of following chart declines as sample size increases

    1 Z_Method

    2 Exponential_Chart

- For every chart, as deviation increases, they perform better
- All charts perform better in positive process mean or process variance deviation than same value of negative process mean or process variance deviation

4.2 Scope of the Future Work

In the the present work an attempt is made to study the performance of different multivariate quality-control charts w r t correlation coefficient, sample size and process mean or variance deviation.

Although these charts have triggering capabilities that process is going out-of-control but only T^2 chart and Z_method have diagnostic procedure for out-of-control variables Others do not have diagnostic procedure, which can be a big problem if numbers of variables are large Multivariate Cusum Charts give good results but its main drawback is the absence of diagnostic procedure for out-of-control signal A study may be done for developing the diagnostic procedure of these charts.

Reference

1 Introduction to Statistical Quality Control, DOUGLAS C MONTGOMERY

2 Multivariate Quality Control, CAMIL FUCHS & RON S KENETT

3 CROSIER R. B (1988) "Multivariate Generalization of Cumulative Sum Quality Control Schemes" Technometrics 30, pp 291-303

4 HAWKINS, D M (1991), "Multivariate Quality Control Based on Regression-Adjusted Variables" Technometrics 33, pp 61-75

5 HAWKINS, D M (1993) " Regression Adjustment for Variables in Multivariate Quality Control" Journal of Quality Technology 25, pp 170-182

6 HOLMES, D S and MERGEN, A E , "Identifying the Source for Out-of-Control Signals When the T2 Control Chart is Used", Quality Engineering, 8(1) pp 137-143 (1995-96)

7 HOTELLING , H (1947) "Multivariate Quality Control - Illustrated by the Air Testing of Sample Bombsights" in Techniques of Statistical Analysis (Eisenhart, C ,Hastay, M W , and Wallis, W A eds) Mcgraw Hill, New York, NY

8 LOWRY, C A , WOODALL, W H , CHAMP, C W , and RIGDON, S E (1992) "A Multivariate Exponentially Weighted Moving Average Control Chart", Technometrics 34, pp 46-53

9 PIGNATIELLO, J J and RUNGER, G C. (1990) "Comparison of Multivariate CUSUM Charts" Journal of Quality Technology 22, pp 173-186

10 TRACY, N D' YOUNG, J C, and MASON, R L (1992) "Multivariate Control Charts for Individual Observations" Journal of Quality Technology 24, pp 88-95

11 WOODALL, W H and NCUBE, M M (19985) "Multivariate CUSUM Quality-Control Procedure" Technometrics 27, pp 285-292

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